

Optimal Admission and Eviction Control of Secondary Users at Cognitive Radio HotSpots

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Abstract—Cognitive Radio (CR) HotSpot is a typical example of commercialized Dynamic Spectrum Access (DSA) in which a Wireless Service Provider (WSP) provides CR end-users access to its local-area network. A WSP temporarily leases licensed channels from the primary licensees via spectrum auction, and subleases them to CR end-users or customers, each demanding a different amount of spectrum, according to its own advertised pricing policy. The CR customers use the subleased channels opportunistically when the channels are not being used by primary/legacy users, and hence, the amount of available spectrum resource varies with time. A WSP should, therefore, maximize its average profit by ‘optimally’ (in some sense) (1) controlling the admission/rejection of arriving CR end-users, and (2) determining a policy of evicting in-service opportunistic users at the channel to which primary users return. To our best knowledge, this is the first attempt to jointly optimize admission and eviction controls for the dynamic spectrum market.

The WSP’s profit maximization problem is formulated with a Semi-Markov Decision Process (SMDP) and its corresponding Linear Programming (LP) setup. This problem is found to become Nonlinear Programming (NLP) subject to two Quality-of-Service (QoS) constraints on request-blocking and user-dropping probabilities, which can fortunately be converted to LP via some manipulation. Using an extensive numerical analysis, we discovered that the derived optimal policy achieves up to 81% more profit than a Complete-Sharing (CS) algorithm in the tested scenario. We also investigate tradeoffs between the two QoS constraints, and consider other important factors affecting WSP’s profit maximization such as the number of leased channels, the end-user pricing, and the cost for reimbursing evicted customers.

I. INTRODUCTION

Dynamic Spectrum Access (DSA) has opened a new way of solving the spectrum scarcity problem that is expected to occur due to the conventional *static* spectrum allocation policy as well as rapidly growing wireless applications and services. The static allocation policy has already reached its limits in finding unallocated spectrum for new emerging wireless services, since there has been an exponential growth of spectrum demand over the past decade or so [1]. DSA is expected to solve, or at least alleviate, this problem by enabling unlicensed *secondary* users (SUs) to opportunistically utilize the licensed spectrum whenever it is left unused by legacy *primary* users (PUs).

The application of DSA ranges from public to commercial and military networks. We will, however, focus on commercial networks, especially spectrum resource allocation to Cognitive

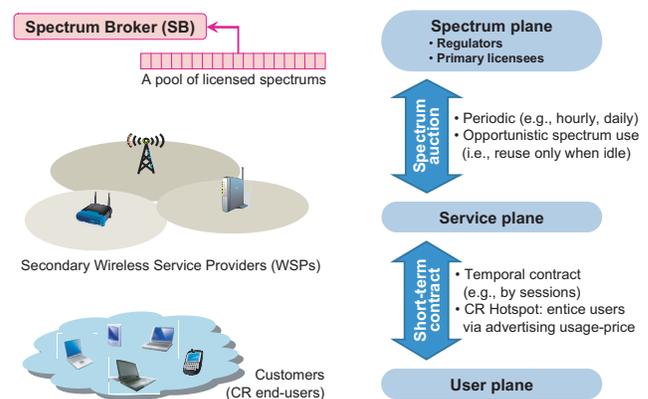


Fig. 1. Inter-plane interactions in the dynamic spectrum market

Radio (CR) users¹ in such networks, since the benefit of DSA can be easily captured by widely spread commercial services.

In a commercial DSA network, the primary license holders may temporarily transfer their spectrum usage rights to CR users (i.e., SUs) [2], [3]. The *Dynamic Spectrum Market* (DSM) is a secondary spectrum market that facilitates the transfer of spectrum rights via an auction mechanism. DSM is composed of three interacting layers/planes as shown in Fig. 1: spectrum plane, service plane, and user plane [3], [4]. The spectrum is auctioned by a Spectrum Broker (SB) at the top plane where bidders are the *secondary* WSPs residing at the middle plane [5], [6]. Here, the SB might be either the regulatory authorities (e.g., FCC in USA and Ofcom in UK) or an authorized third-party. A secondary WSP leases spectrum via auction and subleases a portion of the leased channels to its customers at the bottom plane [7].

A. Motivations

1) *Preemptable Spectrum Leasing:* In [2], Buddhikot introduced four types of dynamic spectrum access—*command and control*, *exclusive-use*, *shared-use*, and *commons*. Of these, the following two types seem to prevail in the literature. First, in the *shared-use* model, SUs are opportunistically access the licensed bands without paying any usage charge to licensees. Their spectrum reuse is allowed as long as the SUs do not

¹Throughout this paper, we will use the terms SUs and CR users interchangeably.

interfere with the PUs. The IEEE 802.22 [8] is an example of such type where SUs opportunistically access TV bands. Although this model may achieve more efficient spectrum utilization, it is not likely to be accepted in the current market since there is no economic incentive for the license holders. Second, in the *dynamic exclusive-use* model, licensees can dynamically lease their spectrum to the lessee (i.e., SU) who temporarily owns an exclusive right to access the spectrum. Although this model can provide extra profit to the licensees, the PUs cannot re-claim their channels before the leasing period ends even if they encounter an unexpected shortage of spectrum.

We propose a new model which is a combination of the above two types by exploiting their advantages. In our model, the license holders can temporarily lease their channels to SUs and generate an additional revenue by charging for their opportunistic use of paid-but-idle channels. The SUs are allowed to use the leased channels only when they are not occupied by the PUs because the licensed users are given priority over the unlicensed CR users. However, the SUs can also benefit from this because they can access the spectrum at a much lower cost than legacy services. In this sense, our model can be considered as a *preemptable* secondary licensing of spectrum.

2) *CR HotSpots Owned by Secondary WSPs*: We view *CR HotSpots* as a typical CR service provided by secondary WSPs in the commercial DSA scenario. They are akin to WLAN HotSpots located at popular public sites like coffee shops, stadiums, libraries, or airports, where a WSP owns CR access points and provides CR end-users wireless services, such as Internet access. The WSP advertises its pricing policy to entice customers, and each CR end-user sets up a short-term contract with the WSP. This type of service may emerge in early stages of DSA deployment due to its similarity to, and familiarity with, the wireless services available today.

3) *Profit Maximization by secondary WSPs*: This paper focuses on how a secondary WSP operating CR HotSpots can maximize its profit by user admission and eviction controls at each CR HotSpot when the spectrum demand varies with CR users and applications, e.g., audio/video users will need more bandwidth than text-only users. This particular application of DSA, in fact, reflects more general optimization issues in various DSA scenarios including public and military networks. The revenue and cost considered in this paper can be generalized as “reward,” with which various types of goal can be expressed and optimized. For example, in the battlefield network, user-admission control may focus on maintaining reliable communication channels within the troops by assigning better quality channels to the commanding officers, where the reward is interpreted as link quality or available bandwidth.

Since the usage charges may differ between the users requesting different amounts of spectrum, accepting user arrivals whenever there is room in the leased channels, called *Complete Sharing* (CS) [9], does not necessarily maximize the WSP’s profit. For example, it is sometimes more profitable for the WSP to reject a user’s request with a small spectrum demand,

even if there is room for it, and to wait until a user with a larger spectrum demand arrives. Thus, the optimal admission control is essential to the profit maximization.

A policy of evicting in-service users also plays an important role in maximizing profit due to PUs’ ON-OFF channel usage patterns. Since PUs are given priority over SUs, the SUs must vacate a channel (called *channel vacation*) when its PUs return to the channel (i.e., the channel state changes from OFF to ON). At the time of channel vacation, the customers previously assigned to the channel have to be relocated by the WSP to the other remaining idle channels. However, if the remaining idle channels cannot fully support spectrum demands of all in-service users,² the WSP should determine which users to be evicted from its network. The evicted users will be compensated with some form of reimbursement, which may differ by the user-specific spectrum demand and thus affect the WSP’s profit.

B. Contributions

We address the problem of determining the optimal user admission and eviction controls using a Semi-Markov Decision Process (SMDP), since user arrivals/departures and channel-state transitions are stochastic. For SMDP, there are a number of well-known efficient algorithms to determine the optimal actions (i.e., admission or eviction) at each decision epoch. Using the basic components of SMDP, a Linear Programming (LP) algorithm is formulated and analyzed. To our best knowledge, this is the first attempt to jointly optimize admission and eviction controls with time-varying spectrum resources.

QoS provisioning for CR end-users is also considered by adding two constraints to the SMDP formulation: the probability of blocking newly-arriving users, and the probability of evicting/dropping in-service users. By making these two QoS metrics upper-bounded, the customers can get some form of QoS assurance and the WSP can strike a balance between profit maximization and customer satisfaction. Although the problem becomes Nonlinear Programming (NLP) by introducing the second QoS metric (i.e., the dropping probability), we show in Section IV that it can be converted to LP by properly manipulating the inequality constraint on the dropping probability.

C. Related Work

Mutlu *et al.* [10] studied the problem of maximizing a WSP’s average profit. However, they focused only on optimizing the pricing policy without considering admission control which could further increase the WSP’s profit. Moreover, they assumed that PUs and SUs can coexist in the same channel (thus unneeding eviction control) which is not always possible since the PUs are given priority over SUs in accessing the licensed channels. Ishibashi *et al.* [11] considered multi-homed PUs, where each PU is either classical or CR-enabled. They investigated enhancement of resource utilization with cognitive PUs switching between channels, and derived the blocking and

²We do not consider QoS degradation (i.e., assigning less bandwidth than a user wants) in this paper.

dropping probabilities in such a scenario. However, no priority in channel access is given to the classical PUs, thus unaccounting for channel eviction. Wang *et al.* [12] proposed a primary-prioritized Markov approach where PUs have exclusive rights to access their own channels. Although the problem setup is somewhat similar to ours, they only considered a single channel and two SUs, thus limiting its applicability. Moreover, they did not consider admission control. Ross and Tsang [9] investigated the problem of optimal admission control when spectrum-demand varies with users. However, channels are assumed always available (i.e., always OFF), and hence, user eviction was not studied.

D. Organization

Section II introduces the system model and basic assumptions used in this paper. In Section III, the problem of maximizing a WSP's profit is modeled as SMDP and its relevant components are derived. Section IV presents an LP-based SMDP algorithm to determine the optimal user admission and eviction policy with two QoS constraints, blocking and dropping probabilities. The proposed scheme is evaluated in Section V via numerical analysis and simulation, and then the paper concludes with Section VI.

II. SYSTEM MODEL

A. Channel Model

As illustrated in Fig. 2, we model a channel as an ON/OFF alternating renewal process [13], denoted by $Z^i(t)$ (i : channel index). This type of channel model has been successfully used in many applications [14]–[16], where the ON (or OFF) period implies the duration with (or without) PUs' signal activities. Hence, SUs are allowed to access the channel only within OFF periods. We assume that ON/OFF transitions can be detected by spectrum sensing, which is not a main focus of this paper. Interested readers may refer to [16] for more details on spectrum sensing.

The sojourn times in ON and OFF states are represented by random variables T_{ON}^i and T_{OFF}^i whose probability density functions (pdfs) are denoted by $f_{T_{ON}^i}(t)$ and $f_{T_{OFF}^i}(t)$, $t > 0$, respectively. ON and OFF states are statistically independent of each other. Although $f_{T_{ON}^i}(t)$ and $f_{T_{OFF}^i}(t)$ can be *any* distribution function, here we consider exponentially-distributed ON and OFF periods with mean $1/\mu_{ON}^i$ and $1/\mu_{OFF}^i$, for mathematical tractability. Extension of our results to general ON/OFF distributions is left as part of our future work.

Let C_i denote the capacity (or bandwidth) of channel i . In this paper, we assume channel capacities are homogeneous, i.e., $C_i = C$, $\forall i$, for ease of illustration. However, our SMDP model can be easily extended to consider heterogeneous channel capacities at the expense of increased state/action spaces.

B. Spectrum Auction Model

We consider a multi-winner periodic spectrum auction [17], [18] where an SB (auctioneer) auctions off the licensed channels periodically (e.g., hourly, daily, or even weekly) every $T_{auction}$, and multiple WSPs bid for the number of channels

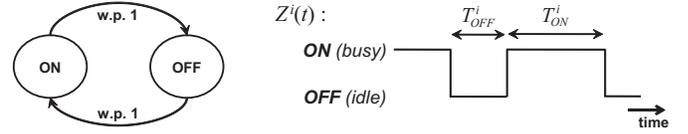


Fig. 2. Channel model: alternating renewal process with ON and OFF states

they want to lease. Once a WSP wins M channels, it pays the (leasing) price $p_{bid}(M)$ per unit-time to secure the temporal rights to reuse the M channels for the period $T_{auction}$, and after $T_{auction}$ the leased channels are returned to the licensees.

We make the following assumption on the bidding cost function $p_{bid}(M)$, $p_{bid}(0) = 0$, which is often accepted in the literature on the CR auction market [7], [19]:

Assumption 1. $p_{bid}(M)$ is a positive, non-decreasing and convex function of M .

This assumption is reasonable because the winning bid is likely to increase faster than proportionally to M due to the competition between WSPs to acquire the limited amount of spectrum resources available in the market. The actual form of $p_{bid}(M)$ should depend on the auction market, and hence we assume $p_{bid}(M)$ is given *a priori* to focus on user-control issues. For an illustrative purpose, $p_{bid}(M) = D_1 \cdot M^{D_2}$, $D_2 \geq 1$, which is introduced in [7], [19] and satisfies Assumption 1, will be used for simulation in Section V where D_2 implies the degree of competition in the auction.

With M (possibly non-contiguous) leased channels, a WSP's total capacity becomes $M \cdot C$. Due to the ON/OFF channel usage patterns, however, its instantaneous capacity is given as $m \cdot C$, $0 \leq m \leq M$, where m is the number of idle channels at that instant. We assume that the pool of idle channels is managed as one *logical channel*, whose capacity is equal to $m \cdot C$ bandwidth-units. This is achievable via Orthogonal Frequency Division Multiplexing (OFDM) techniques with adaptive and selective allocation of OFDM sub-carriers, like NC-OFDM (Non-Contiguous OFDM) proposed in [20].

C. Multi-class User QoS Model

Customers are the CR-enabled end-devices seeking a secondary WSP that can best serve their needs. Each customer is assumed to have a spectrum demand which falls into one of the following K QoS classes:

$$\mathbf{B} = (B_1, B_2, \dots, B_K)^T,$$

where B_k is the bandwidth requirement of class- k customers³ and T represents 'transpose'. Without loss of generality, we can assume $B_{min} = B_1 < B_2 < \dots < B_K = B_{max}$.

Based on selective OFDM sub-carrier allocation, each CR end-user is assumed to be capable of tuning its antenna to any portion of the logical channel for its bandwidth assignment of B_k , as illustrated in Fig. 3. In this way, at a channel's OFF→ON transition, the users in the channel can be redistributed to other idle channels, by updating the mapping of

³We reserve k and i as *class* and *channel* indexes, respectively.

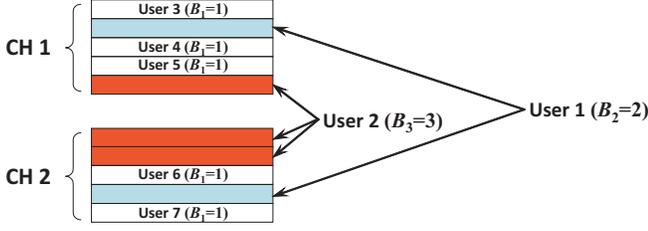


Fig. 3. An example of channel allocation with $M = 2$, $K = 3$, and $C = 5$.

OFDM subcarriers to the users. MAC-layer beaconing might be used to perform this re-mapping in real time.

D. User Traffic Model

We assume that the arrival of class- k customers follows a Poisson process with rate λ_k . As shown in [21], user-oriented connection requests are modeled well as a Poisson arrival process. Since the requests from the CR users are connection-level, this assumption captures reasonably well the reality of the secondary market. For mathematical tractability, the service time of an admitted class- k customer is assumed exponentially distributed with mean $1/\mu_k$. This assumption captures some reality of wireless applications such as phone-call traffic with exponentially-distributed talk spurt and silence periods [22].

E. End-User Pricing Model

The revenue of a WSP is generated from the CR end-users who pay fees for their opportunistic spectrum usage. The vector of the usage fees for K QoS-classes, in terms of price *per unit-time per unit-bandwidth*, is denoted as

$$\mathbf{p} = (p_1, p_2, \dots, p_K)^T,$$

where p_k could be either constant, non-decreasing, or non-increasing in k , depending on the WSP's pricing policy.

The arrival rate is in fact price-dependent, and therefore, it is denoted by $\lambda_k(p_k)$. $\lambda_k(p_k)$ is a non-increasing function of p_k , since a WSP advertising higher prices should expect less customer arrivals than its competitors offering lower prices. The actual price-arrival relationship depends highly on the WSP's tariff and the degree of competition, which must be treated as a separate marketing research problem. Here we assume $\lambda_k(p_k)$ is given *a priori* and confine our discussion to optimal user control. We leave $\lambda_k(p_k)$ as a general parameter in the analysis, and we will use the following example in Section V for an illustrative purpose: $\lambda_k(p_k) = \lambda_k^{max} e^{-\delta_k p_k}$, as introduced in [23]. Here, λ_k^{max} indicates that the maximum user population at a CR HotSpot is bounded, and δ_k represents the rate of decrease of the arrival rate as p_k increases, which is related to the degree of competition between WSPs.

III. SMDP FORMULATION

The profit maximization at each CR HotSpot faces a unique challenge due to time-varying channel availability. Therefore, a secondary WSP must jointly consider user admission and

eviction controls to maximize its profit subject to some QoS constraints, such as keeping the blocking and dropping probabilities within an acceptable range.

We formulate SMDP by accounting for time-varying channel availability. The basic and essential components of the SMDP are derived by considering possible actions to be taken for the joint control of user admission and eviction.

A. System State and State Space

We start with the definition of system state as

$$\mathbf{s} = (\mathbf{n}, \mathbf{w}), \quad \begin{cases} \mathbf{n} = (n_1, n_2, \dots, n_K)^T, \\ \mathbf{w} = (w_1, w_2, \dots, w_M)^T, \end{cases}$$

where n_k is the number of class- k customers in service, and w_i is the channel state defined as:

$$w_i = \begin{cases} 1, & \text{channel } i \text{ is occupied by primaries (i.e., ON),} \\ 0, & \text{channel } i \text{ is not occupied by primaries (i.e., OFF).} \end{cases}$$

The set of possible sub-states \mathbf{n} for a given channel state \mathbf{w} is thus determined as

$$\Lambda_{\mathbf{n}|\mathbf{w}} = \{\mathbf{n} : n_k \geq 0, \mathbf{n}^T \mathbf{B} \leq mC, m = (\mathbf{1} - \mathbf{w})^T \mathbf{1}\},$$

where m is the number of *idle* channels as defined earlier, and $\mathbf{1}$ is a vector of 1's. In addition, the set of possible sub-states \mathbf{w} is given as

$$\Lambda_{\mathbf{w}} = \{\mathbf{w} : w_i \in \{0, 1\}\}.$$

Therefore, the state space Λ is given as

$$\Lambda = \{\mathbf{s} : \mathbf{w} \in \Lambda_{\mathbf{w}}, \mathbf{n} \in \Lambda_{\mathbf{n}|\mathbf{w}}\}.$$

B. Possible Actions and Action Space

In our SMDP formulation, an *action* is taken and updated at each *decision epoch* under the chosen policy, where a natural choice of the decision epoch is the instant when system state changes, i.e., (1) at class- k customer's arrival/departure and (2) at channel i 's state transition (ON→OFF or OFF→ON).

We define the action at a certain decision epoch as

$$\boldsymbol{\alpha} = (\mathbf{a}, \mathbf{b}), \quad \begin{cases} \mathbf{a} = (a_1, a_2, \dots, a_K)^T, \\ \mathbf{b} = (b_1, b_2, \dots, b_K)^T, \end{cases}$$

where \mathbf{a} is the admission policy for *future* customer arrivals, such that

$$a_k = \begin{cases} 0, & \text{reject all future class-}k \text{ arrivals,} \\ 1, & \text{admit all future class-}k \text{ arrivals,} \end{cases}$$

and \mathbf{b} is the eviction policy of *in-service* customers with b_k indicating the number of class- k customers to be evicted *at the time of channel vacation*. Note that \mathbf{b} is effective only when a channel vacation happens (i.e., at OFF→ON of a certain channel). That is, even if $\mathbf{b} \neq \mathbf{0}$, we do not evict any customer if the next event is arrival, departure, or ON→OFF.

Then, for a given state \mathbf{s} , the action space A is defined as

$$\begin{aligned} A_1(\mathbf{s}) &= \{ \mathbf{a} : a_k \in \{0, 1\}; a_k = 0 \text{ if } (\mathbf{n} + \mathbf{u}_k)^T \mathbf{B} > mC \}, \\ A_2(\mathbf{s}) &= \{ \mathbf{b} : 0 \leq b_k \leq n_k, (\mathbf{n} - \mathbf{b})^T \mathbf{B} \leq (m-1)C, \\ &\quad (\mathbf{n} - \mathbf{b} + \mathbf{u}_k)^T \mathbf{B} > (m-1)C \text{ for } \forall k \text{ s.t. } b_k \neq 0; \\ &\quad \mathbf{b} = \mathbf{0} \text{ if } \mathbf{n}^T \mathbf{B} \leq (m-1)C \}, \\ A(\mathbf{s}) &= \begin{cases} \{ \boldsymbol{\alpha} : \mathbf{a} = \mathbf{0}, \mathbf{b} = \mathbf{0} \}, & \text{if } m = 0, \\ \{ \boldsymbol{\alpha} : \mathbf{a} \in A_1(\mathbf{s}), \mathbf{b} \in A_2(\mathbf{s}) \}, & \text{otherwise,} \end{cases} \end{aligned}$$

where \mathbf{u}_j is a unit vector with a single 1 at the j -th position and 0's elsewhere. In the definition of $A_2(\mathbf{s})$, the constraint

$$(\mathbf{n} - \mathbf{b} + \mathbf{u}_k)^T \mathbf{B} > (m-1)C \text{ for } \forall k \text{ s.t. } b_k \neq 0$$

implies that \mathbf{b} should be *minimal* so as not to evict more in-service customers than necessary. That is, \mathbf{b} is *not* minimal if there exists k such that $\mathbf{b}' = \mathbf{b} - \mathbf{u}_k$ and $(\mathbf{n} - \mathbf{b}')^T \mathbf{B} \leq (m-1)C$, i.e., evicting one less customer than \mathbf{b} still fits in $(m-1)$ channels. Obviously, the choice of minimal \mathbf{b} for a given \mathbf{s} is not unique. For example, in case $m = 2$, $C = 5$ and $\mathbf{B} = (1, 2)$, there are three possible minimal \mathbf{b} 's for $\mathbf{n} = (5, 2)$: $\mathbf{b} = (4, 0)$, $\mathbf{b} = (2, 1)$ and $\mathbf{b} = (0, 2)$.

C. Eligibility for SMDP Formulation

Before deriving other essential components of the SMDP, we need to check if the system under consideration can be modeled as SMDP. In [24], SMDP is defined as follows.

Definition 1. A dynamic system is said to be a semi-Markov decision process if the following property is satisfied: if at a decision epoch the action $\boldsymbol{\alpha}$ is chosen in state \mathbf{s} , then the time until, the state at, and revenue/cost incurred until the next decision epoch depend only on the current state \mathbf{s} and its chosen action $\boldsymbol{\alpha}$.

According to our assumption, inter-arrival and inter-departure times of class- k customers are all exponentially distributed. Since the arrival and departure processes are independent of each other, the combined random process is still exponentially distributed with mean $1/(\mathbf{n}^T \boldsymbol{\mu} + \mathbf{a}^T \boldsymbol{\lambda})$, where $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_K)^T$, and $\boldsymbol{\mu} = (\mu_1, \mu_2, \dots, \mu_K)^T$. Second, since channel i 's residual time in ON (or OFF) state is exponentially distributed with mean $1/\mu_{ON}^i$ (or $1/\mu_{OFF}^i$), channel i 's state transition process becomes exponentially distributed with mean $1/((1-w_i)\mu_{OFF}^i + w_i\mu_{ON}^i)$. Therefore, with M independent channels, the combined random process is also exponentially distributed with mean $[(\mathbf{1} - \mathbf{w})^T \boldsymbol{\mu}_{OFF} + \mathbf{w}^T \boldsymbol{\mu}_{ON}]^{-1}$, where $\boldsymbol{\mu}_{OFF} = (\mu_{OFF}^1, \mu_{OFF}^2, \dots, \mu_{OFF}^M)^T$ and $\boldsymbol{\mu}_{ON} = (\mu_{ON}^1, \mu_{ON}^2, \dots, \mu_{ON}^M)^T$. Finally, the combination of all four possible events—arrival and departure (class- k), and ON→OFF and OFF→ON transitions (channel i)—becomes exponentially distributed with mean

$$[\mathbf{n}^T \boldsymbol{\mu} + \mathbf{a}^T \boldsymbol{\lambda} + (\mathbf{1} - \mathbf{w})^T \boldsymbol{\mu}_{OFF} + \mathbf{w}^T \boldsymbol{\mu}_{ON}]^{-1}. \quad (1)$$

As a result, the system model considered in this paper can be modeled as SMDP, since it possesses the necessary properties in Definition 1 thanks to the memoryless property of an exponential distribution.

D. Decision Epochs

Since the combined process of four possible events is exponentially distributed with mean as in Eq. (1), the expected time between two decision epochs, denoted by $\tau_{\mathbf{s}}(\boldsymbol{\alpha})$, is determined as

$$\tau_{\mathbf{s}}(\boldsymbol{\alpha}) = [\mathbf{n}^T \boldsymbol{\mu} + \mathbf{a}^T \boldsymbol{\lambda} + (\mathbf{1} - \mathbf{w})^T \boldsymbol{\mu}_{OFF} + \mathbf{w}^T \boldsymbol{\mu}_{ON}]^{-1}.$$

E. State-Transition Probability

The probability that the state of the system switches from $\mathbf{s} = (\mathbf{n}, \mathbf{w})$ to $\mathbf{s}' = (\mathbf{n}', \mathbf{w}')$ at the next decision epoch is

$$p_{\mathbf{s}, \mathbf{s}'}(\boldsymbol{\alpha}) = \begin{cases} a_k \lambda_k \tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n} + \mathbf{u}_k, \mathbf{w}' = \mathbf{w}, \\ & \text{(class-} k \text{ arrival),} \\ n_k \mu_k \tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n} - \mathbf{u}_k, \mathbf{w}' = \mathbf{w}, \\ & \text{(class-} k \text{ departure),} \\ \mu_{OFF}^i \tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n} - \mathbf{b}, \mathbf{w}' = \mathbf{w} + \mathbf{u}_i, \\ & \text{(channel } i \text{: OFF} \rightarrow \text{ON),} \\ \mu_{ON}^i \tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n}, \mathbf{w}' = \mathbf{w} - \mathbf{u}_i, \\ & \text{(channel } i \text{: ON} \rightarrow \text{OFF),} \\ 0, & \text{(otherwise).} \end{cases}$$

F. Revenue and Reimbursement Cost

Let $r_{\mathbf{s}}(\boldsymbol{\alpha})$ and $c_{\mathbf{s}}(\boldsymbol{\alpha})$ denote the expected revenue and the cost incurred by customers until the next decision epoch if action $\boldsymbol{\alpha}$ is chosen at state \mathbf{s} , respectively. Since the revenue comes from the usage fee paid by the admitted customers, $r_{\mathbf{s}}(\boldsymbol{\alpha})$ is given as

$$r_{\mathbf{s}}(\boldsymbol{\alpha}) = \sum_k p_k B_k n_k \tau_{\mathbf{s}}(\boldsymbol{\alpha}).$$

Assuming a fixed amount of reimbursement I_k for an evicted class- k customer, $c_{\mathbf{s}}(\boldsymbol{\alpha})$ is

$$\begin{aligned} c_{\mathbf{s}}(\boldsymbol{\alpha}) &= \sum_k I_k b_k \cdot q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha}), \\ q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha}) &= \sum_{\substack{\mathbf{s}' \in \Lambda \text{ s.t. } \mathbf{n}' = \mathbf{n} - \mathbf{b}, \\ \mathbf{w}' = \mathbf{w} + \mathbf{u}_i \text{ for some } i}} p_{\mathbf{s}, \mathbf{s}'}(\boldsymbol{\alpha}), \end{aligned}$$

where $\tau_{\mathbf{s}}(\boldsymbol{\alpha})$ does not contribute to the equation since the reimbursement is a one-time cost at channel vacation. $q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha})$ in the above equation implies the probability that the event of channel vacation will happen at the next decision epoch. We also let $q_{\mathbf{s}}^{\mathcal{D}}(\boldsymbol{\alpha})$ denote the probability that the event of class k departure will happen at the next decision epoch, which is defined as:

$$q_{\mathbf{s}}^{\mathcal{D}}(\boldsymbol{\alpha}) = \sum_{\substack{\mathbf{s}' \in \Lambda \text{ s.t.} \\ \mathbf{n}' = \mathbf{n} - \mathbf{u}_k, \mathbf{w}' = \mathbf{w}}} p_{\mathbf{s}, \mathbf{s}'}(\boldsymbol{\alpha}).$$

IV. OPTIMAL USER CONTROL VIA AN SMDP ALGORITHM

So far we have defined and derived all necessary components of SMDP. Now, we want to find the optimal action $\boldsymbol{\alpha}$ for each possible system state \mathbf{s} to maximize the average profit, using the derived SMDP components. There are three well-known methods for optimally solving the SMDP problem: policy-iteration, value-iteration, and Linear Programming (LP)

[24]. Of these, LP is the best known method to model the QoS-constrained optimization problem thanks to its flexibility to include additional equality and inequality constraints. Therefore, we formulate an LP problem for the given SMDP with the two QoS-constraints.

A. Linear Programming SMDP Algorithm: Constrained QoS

Here we formulate a three-step LP algorithm with the two given QoS constraints: the probability of blocking class- k arrivals (denoted by P_b^k), and the probability of evicting class- k in-service customers (denoted by P_d^k). This 3-step algorithm follows the general format of the LP algorithm recommended in [24], but its mathematical contents are our own development.

Step 1: Find the optimal basic solution $z_{s,\alpha}^*$ to the following linear programming ($z_{s,\alpha} \geq 0$):

$$\begin{aligned} & \text{Maximize} \quad \sum_{s \in \Lambda} \sum_{\alpha \in A(s)} (r_s(\alpha) - c_s(\alpha)) z_{s,\alpha} \\ & \text{Subject to} \quad \sum_{\alpha \in A(s')} z_{s',\alpha} - \sum_{s \in \Lambda} \sum_{\alpha \in A(s)} p_{s,s'}(\alpha) z_{s,\alpha} = 0, \quad s' \in \Lambda, \\ & \quad \quad \quad \sum_{s \in \Lambda} \sum_{\alpha \in A(s)} \tau_s(\alpha) z_{s,\alpha} = 1, \\ & \quad \quad \quad P_b^k \leq \gamma_{block}^k, \quad \forall k, \\ & \quad \quad \quad P_d^k \leq \gamma_{drop}^k, \quad \forall k, \end{aligned}$$

where γ_{block}^k and γ_{drop}^k are the upper-bounds on P_b^k and P_d^k . In addition, P_b^k and P_d^k are expressed as follows:

$$\begin{aligned} P_b^k &= \sum_{s \in \Lambda} \sum_{\substack{\alpha \in A(s) \\ \text{s.t. } a_k=0}} \tau_s(\alpha) z_{s,\alpha}, \\ P_d^k &= \frac{\mathcal{V}}{\mathcal{D} + \mathcal{V}}, \quad \begin{cases} \mathcal{D} = \sum_{s \in \Lambda} \sum_{\alpha \in A(s)} \tau_s(\alpha) z_{s,\alpha} q_s^{\mathcal{D}}(\alpha) \cdot 1, \\ \mathcal{V} = \sum_{s \in \Lambda} \sum_{\alpha \in A(s)} \tau_s(\alpha) z_{s,\alpha} q_s^{\mathcal{V}}(\alpha) \cdot b_k, \end{cases} \end{aligned}$$

where $z_{s,\alpha} = x_{s,\alpha} / \tau_s(\alpha)$, and $x_{s,\alpha}$ denotes the fraction of time that the system is in state s when action α is chosen. Therefore, \mathcal{D} implies the expected number of departure until the next decision epoch and \mathcal{V} implies the expected number of evictions until the next decision epoch.

Due to the form of P_d^k , this becomes a Nonlinear Programming (NLP) problem. Fortunately, however, by properly manipulating the constraint $P_d^k \leq \gamma_{drop}^k$, it can be converted to a Linear Programming (LP) problem as follows:

$$\begin{aligned} P_d^k &= \frac{\mathcal{V}}{\mathcal{D} + \mathcal{V}} \leq \gamma_{drop}^k \Rightarrow (1/\gamma_{drop}^k - 1)\mathcal{V} \leq \mathcal{D} \Rightarrow \\ & \sum_{s \in \Lambda} \sum_{\alpha \in A(s)} \left(q_s^{\mathcal{V}}(\alpha) \frac{1 - \gamma_{drop}^k}{\gamma_{drop}^k} \cdot b_k - q_s^{\mathcal{D}}(\alpha) \right) \tau_s(\alpha) z_{s,\alpha} \leq 0, \end{aligned}$$

which is a linear constraint on $z_{s,\alpha}$'s.

Step 2: Start with a non-empty set

$$S := \left\{ s \mid \sum_{\alpha \in A(s)} z_{s,\alpha}^* > 0 \right\},$$

and for any state $s \in S$, set the decision as

$$R^*(s) := \alpha \text{ for some } \alpha \text{ such that } z_{s,\alpha}^* > 0.$$

Step 3: If $S = \Lambda$, then the algorithm terminates with the optimal policy R^* . Otherwise, determine some state $s \notin S$ and action $\alpha \in A(s)$ such that $p_{s,s'}(\alpha) > 0$ for some $s' \in S$. For the chosen s , set $R^*(s) := \alpha$ and update $S := S \cup \{s\}$, and then repeat Step 3.

By repeatedly performing Step 3, the algorithm runs until S becomes Λ . The complexity of executing this final step is trivial since $p_{s,s'}(\alpha) > 0$ has already been computed in Step 1 for all possible combinations of (s, s', α) . In addition, the optimality of the derived policy is guaranteed in [24], although the algorithm may not produce a unique solution due to the conditions ‘some α ’ and ‘some s ’ in step 2 and 3.

Then, the *optimal profit g^* per unit-time* is determined as

$$g^* = \sum_{\substack{s \in \Lambda \\ \alpha = R^*(s)}} (r_s(\alpha) - c_s(\alpha)) z_{s,\alpha} - \text{pbid}(M). \quad (2)$$

B. Convergence of the SMDP algorithm

The convergence of the above algorithm must be shown to ensure that it completes in a finite number of steps. Its convergence can be proven by showing that the considered MDP is *unichain* [25], where ‘‘unichain’’ is defined as follows.

Definition 2. An MDP is *unichain* if it consists of a single recurrent class plus a (possibly empty) set of transient states for every stationary policy R . [25]

Definition 3. A *recurrent class* is a communication class such that no state outside the class is accessible from any state inside it [26].

We now prove that the MDP considered here is unichain.

Proposition 1. The SMDP considered here is unichain, and thus the proposed LP algorithm converges to the optimal stationary policy in a finite number of steps.

Proof: Suppose there exist more than one recurrent class, and assume the state $s_0 = (\mathbf{0}, 1)$ is in a recurrent class S_0 . Then, for any $s \notin S_0$, s_0 should not be accessible from s by Definition 3. However, s_0 is in fact accessible from s within $m = (\mathbf{1} - \mathbf{w})^T \mathbf{1} \leq M$ transitions with probability $p_{s,s}^{(1)} p_{s,s}^{(2)} \cdots p_{s,s}^{(m-1)} > 0$, if we choose the states $\{s^{(1)}, s^{(2)}, \dots, s^{(m-1)}\}$ such that each transition makes one of the m idle channels occupied while already occupied channels remain occupied. This is a contradiction. Therefore, there exists only one recurrent class, and the SMDP is thus unichain. ■

C. Complexity of SMDP Algorithm

The complexity of the proposed algorithm is measured by the size of the search space $\Lambda \times A(s)$. One can easily see that $|\Lambda \times A(s)|$ increases exponentially as M or K increases, because $|\Lambda_w| = 2^M$ and $|A_1(s)| = 2^K$ in the worst case. Note, however, that the LP can be solved off-line before a WSP starts a new spectrum leasing period (i.e., T_{auction}), so the derived optimal control can be stored in a database. Using the database, the WSP can perform user admission control in real time by simply looking up the database at every user arrival/departure or channel ON/OFF transition.

V. PERFORMANCE EVALUATION

In this section, the performance of the proposed SMDP algorithm is evaluated, and its sensitivity to M , \mathbf{p} , and I_k (reimbursement cost) is studied via a series of numerical analyses and simulations.

First, Section V-A presents the state transition diagram of the proposed SMDP algorithm and compares its optimal profit with the profit by the CS (Complete Sharing) algorithm [9]. Then, in Section V-B, we discuss how the blocking and dropping probabilities affect each other. In Section V-C, we study how sensitive the optimal profit is to the end-user pricing \mathbf{p} . Finally, Section V-D discusses the relationship between the reimbursement cost and the optimal profit.

In all tests, simulation is performed by randomly generating Poisson arrivals/departures (as assumed in Section II) and exponentially-distributed ON/OFF durations. Each decision epoch takes an action in accordance with the set of optimal actions determined by the analysis. A simulation ran for 3,000 time units and the same simulation repeats 10 times to get its average performance.

The simulation parameters are set as follows.

1) Parameters common to all tests are

- $C = 5$, and $B_k = k, \forall k; p_{\text{bid}}(M) = \frac{1}{15} M^{2.5}$,
- $E[T_{OFF}^i] = 1, E[T_{ON}^i] = 0.5, \forall i$,
- $\lambda_k(p_k) = \lambda_k^{\text{max}} e^{-p_k}$, where $(\lambda_1^{\text{max}}, \lambda_2^{\text{max}}, \lambda_3^{\text{max}}) = (\frac{1}{4}, 1, \frac{3}{4})$,
- $I_k = \epsilon_I \times p_k B_k / \mu_k, 0 \leq \epsilon_I \leq 1$,

where T_{OFF}^i and T_{ON}^i are all exponentially distributed, and I_k is set to be $\epsilon_I \cdot 100\%$ of the average usage charge until departure (not eviction) for each class- k user.

2) Test-specific parameters are shown in Table I, where $\gamma_{\text{block}} = \gamma_{\text{block}}^k$ and $\gamma_{\text{drop}} = \gamma_{\text{drop}}^k$ for all k . In addition, the end-user price \mathbf{p} , except in Section V-C, is given as

- $p_k = 2 \times 0.7^{k-1}$,

so that $(k+1)$ -th class users get 30% discounted *per* unit-bandwidth price compared to the k -th class users. This type of pricing is frequently used in the actual market where customers using higher quality service (i.e., larger B_k), hence paying more usage fee of $p_k \cdot B_k$, are offered a discounted price. In Section V-C, p_k will be defined separately to show the impact of varying prices on g^* .

Section	M	K	γ_{block}	γ_{drop}	$\boldsymbol{\mu}$	ϵ_I
V-A1	2	2	1.0	1.0	$(1, \frac{1}{2})$	0.5
V-A2	1-5	3	1.0	1.0	$(1, \frac{1}{2}, \frac{1}{4})$	0.5
V-B (a)	2	3	1.0	0.37-0.46	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	0.5
V-B (b)	2	3	0.24-0.33	1.0	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	0.5
V-C	2	2	0.45	0.45	$(1, \frac{1}{2})$	0.5
V-D	2	2	0.45	0.45	$(1, \frac{1}{2})$	0.1-1.0

TABLE I
THE LIST OF TEST-SPECIFIC PARAMETERS

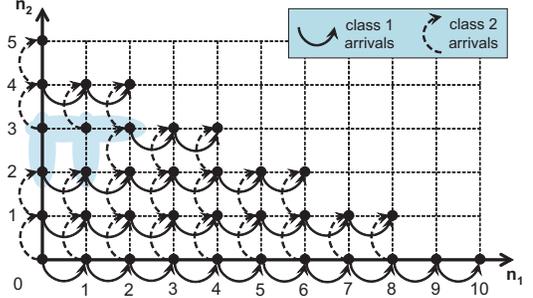


Fig. 4. The state-transition diagrams of the proposed SMDP algorithm, according to user arrivals (shown for $m = 2$)

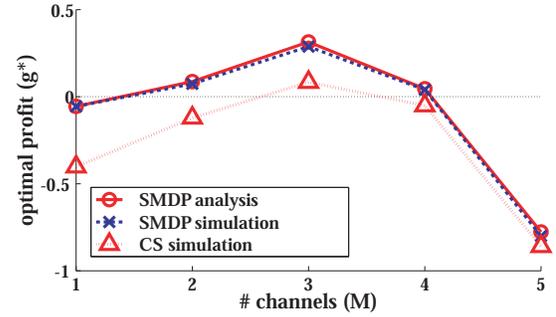
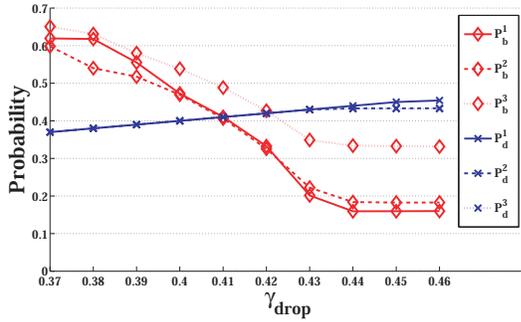


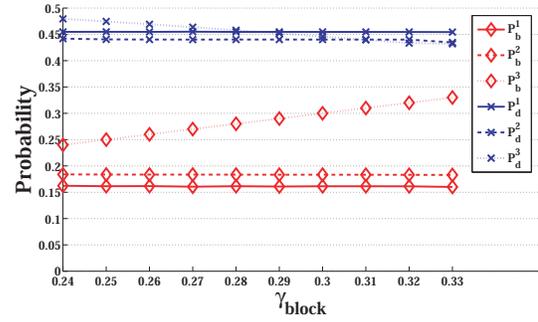
Fig. 5. Comparison of the profits achieved by SMDP and CS

A. Optimal Control and Profit by the Proposed Algorithm

1) *State Transition Diagram*: Fig. 4 illustrates the optimal actions derived by the proposed SMDP algorithm for $M = 2$ and $K = 2$, in the form of a state transition diagram. For simplicity, the state transition diagram is drawn for $m = 2$ (i.e., when there are two idle channels) and only the transitions by the user arrivals are presented. The first thing we notice is that the optimal admission policy deliberately rejects some arrivals to maximize profit, and hence the following state transitions do not occur while they are allowed by the CS algorithm: $(0,2) \rightarrow (0,3)$, $(0,3) \rightarrow (1,3)$, $(1,2) \rightarrow (1,3)$, and $(1,3) \rightarrow (2,3)$. Second, it is known that if the M channels are always available, the optimal admission policy for $K = 2$ shows the threshold behavior [27]. However, we consider time-varying spectrum resource according to ON/OFF channel patterns, and interestingly it turns out that the optimal policy in this case does not follow the threshold behavior. This unique feature is observed partly because the states of $m = 2$ drawn in Fig. 4 can make transition *not only* within the states of $m = 2$ *but also* to the states of $m = 1$ when channel vacation occurs in either of the two channels.



(a) Varying γ_{drop}



(b) Varying γ_{block}

Fig. 6. The tradeoffs between P_b and P_d

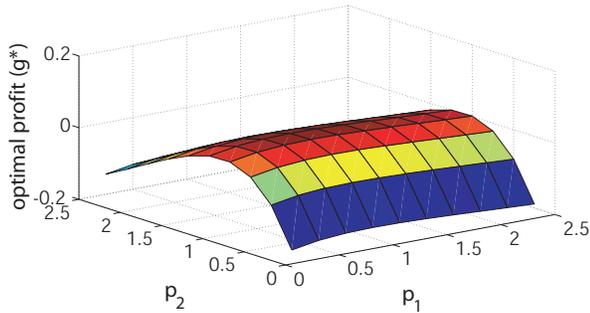


Fig. 7. Optimal profit with various end-user pricing

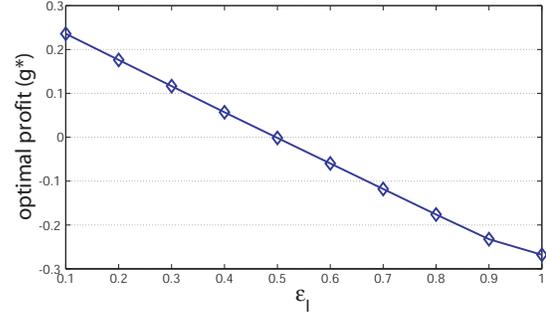


Fig. 8. Optimal profit with different reimbursement cost

2) *Optimal Profit g^** : We compare the performance of the proposed SMDP algorithm with the CS algorithm, in terms of g^* . In general, CS cannot achieve the maximal profit because it does not differentiate users with different spectrum demands. Moreover, the CS algorithm only provides a *passive* admission control with which a user is rejected only when there is no room in the WSP’s resources. Since the traditional CS algorithm is not designed to deal with user evictions, we assume that it randomly chooses users to evict so that the remaining users can be packed into the idle channels. Since CS does not support any QoS guarantee, we use $\gamma_{block} = \gamma_{drop} = 1.0$ for fair comparison.

Fig. 5 shows that SMDP always achieves more profit than CS. For SMDP, the simulation results conform to the analytically-predicted g^* from Eq. (2), indicating that the proposed SMDP model in Section IV was properly analyzed and applied. One can also see that g^* varies dramatically as M changes. For example, g^* is increasing when $M \leq 3$ and decreasing when $M \geq 3$. Interestingly, sometimes g^* even becomes negative. This phenomenon stems from the tradeoff between the revenue generated by customers (i.e., $g^* + p_{bid}(M)$) and the bidding cost (i.e., $p_{bid}(M)$) because— (1) a larger M induces more revenue and less reimbursement cost due to more room available to accommodate user arrivals, but (2) the gain will eventually be saturated due to the bounded user population, and therefore, leasing more channels than necessary becomes unproductive, considering the fact that $p_{bid}(M)$ grows faster than proportionally to M . Therefore,

finding a proper M is essential to maximizing the profit of a WSP. Since our proposed method is to achieve maximal profit g^* at a given M , a WSP may use our method to find g^* for each M and then to determine the proper number of channels to bid (i.e., M) for maximal g^* .

B. Tradeoffs Between Two QoS Constraints

Fig. 6 plots the tradeoff between P_b and P_d . To show the relationship between the two, we first fix $\gamma_{block} = 1.0$ and vary γ_{drop} in (a). Similarly, in (b), we fix $\gamma_{drop} = 1.0$ and vary γ_{block} .

As γ_{drop} decreases, the QoS requirement on P_d becomes stricter in such a way that more in-service users should be protected from eviction at channel vacation. For this, user arrivals must be blocked more often since the longer stay of in-service users implies less idle resources for new arrivals. As a result, P_b increases to compensate for the decrease of P_d , which can be observed from Fig. 6(a).

By contrast, as γ_{block} decreases, the QoS requirement on P_b becomes stricter so that less users are rejected upon their arrival. To accommodate more users, the WSP would have to reserve more room by evicting more users at channel vacation, and thus increasing P_d . However, as seen in Fig. 6(b), there is only a slight increase in P_d as γ_{block} decreases. This is rooted at the fact that an action \mathbf{b} is chosen to be “minimal,” and hence no more users than necessary are evicted. One can make P_d more sensitive to γ_{block} by introducing non-minimal \mathbf{b} ’s in $A_2(\mathbf{s})$ at the cost of an enlarged action space.

C. Impact of End-User Pricing on g^*

Due to the price-dependent arrival rate, i.e., $\lambda_k(p_k) = \lambda_k^{max} e^{-p_k}$, varying p_k would produce different profits. To show its impact on g^* , we tested the case of $M = 2$ and $K = 2$, while varying p_1 and p_2 such as $0.25 \leq p_1, p_2 \leq 2.50$. Unlike other tests, all possible combinations of (p_1, p_2) are considered. Fig. 7 plots the resulting g^* . It can be seen that as p_k gets closer to 0.25 and 2.50, the resultant profit decreases due to the bounded user arrival by λ_k^{max} and decrease of the user arrival rate by e^{-p_k} , respectively. As a result, the profit function becomes concave, where the largest profit can be attained at $p_1 = 1.0, p_2 = 1.0$ in the figure. With the tested scenario, profit change is more sensitive to p_2 because $\lambda_2^{max} > \lambda_1^{max}$ and $1/\mu_2 > 1/\mu_1$. Therefore, a WSP must consider the impact of its pricing policy on the overall profit, for which marketing research is essential to discover the price-arrival rate relationship.

D. Impact of Reimbursement on g^*

Fig. 8 presents the impact of reimbursement policy on g^* , where ϵ_I determines how much of the usage charge should be provided for evicted users as a compensation. The plot clearly shows how sensitive the optimal profit is to the reimbursement cost, where g^* drops almost linearly as ϵ_I increases. Therefore, ϵ_I should be carefully determined so that customer satisfaction is improved while profit is maximized.

VI. CONCLUSION

In this paper, we proposed the optimal admission and eviction controls of CR end-users to maximize the profit of a secondary WSP at each CR HotSpot. The optimization problem was modeled as a Semi-Markov Decision Process (SMDP) and a linear programming algorithm is formulated to derive the optimal actions to be taken upon each user arrival and channel vacation. The two constraints on the user QoS, the probabilities of user blocking and dropping, have also been considered to strike a balance between profit maximization and user satisfaction. The proposed LP algorithm is shown to outperform the complete-sharing resource allocation, and the sensitivity of the optimal profit to three important factors, the number of channels, pricing policy, and reimbursement cost, has also been studied.

In future, we would like to extend the problem to cover cross-layer optimizations that account for the interactions between spectrum, service, and user planes simultaneously. Solutions to this problem will provide a real-time decision mechanism for spectrum auction and service provisioning.

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