Preempt a Job or Not in EDF Scheduling of Uniprocessor Systems

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APPENDIX

In this section, we prove Lemma 2: the time-complexity of Theorem 1 for a given task set with given \( \{X_i\} \) is pseudo-polynomial in the task parameters, if \( \sum_{\tau \in T} C_i/T_i + \sum_{\tau \in T} (C_i + \alpha)/T_i \) is upper-bounded by a constant that is strictly smaller than 1.

We first prove that we need not test Eq. (9) for some \( l \) larger than a certain value. To do this, we present a relevant property of the fp-EDF analysis without any preemption delay [4] as follows:

**Lemma 6**: (Theorem 6 in [28]) Suppose \( U < 1 \) holds and the condition for the fp-EDF analysis without any preemption delay [4] (i.e., Eq. (2) in this paper) is violated for some \( l > 0 \). Then, the condition should be also violated for some \( 0 < l \leq l_{\max} \triangleq \max \{ \max_{\tau \in T} (D_i - T_i) \cdot U_i/(1 - U), \sum_{\tau \in T} (C_i - T_i) \cdot U_i/(1 - U) \} \), where \( U_i \triangleq C_i/T_i \) and \( U \triangleq \sum_{\tau \in T} U_i \).

The lemma implies that we need to test Eq. (2) only for \( 0 < l \leq l_{\max} \). Then, using Lemma 6, we can upper-bound \( l \) for Theorem 1 as follows.

**Lemma 7**: Suppose \( U' < 1 \) holds and Eq. (9) is violated for some \( l > 0 \). Then, the condition is also violated for some \( 0 < l \leq l'_{\max} \triangleq \max \{ D_{\max} \cdot \max_{\tau \in T} (D_i - T_i) \cdot U_i/(1 - U'), \sum_{\tau \in T} (C_i - T_i) \cdot U_i/(1 - U') \} \), where \( U'_i \triangleq C_i/T_i \) for \( X_i = 0 \) and \( U'_i \triangleq (C_i + \alpha)/T_i \) for \( X_i = 1 \), and \( U' \triangleq \sum_{\tau \in T} U'_i \).

**Proof**: Consider a new task set \( T' \) in which all task parameters are the same as \( T \), but the execution time of each \( \tau_i \) with \( X_i = 1 \) is \( C_i + \alpha \). We consider two cases: (i) Eq. (2) for \( T' \), and (ii) Eq. (9) for \( T \). Since \( B \) in Eq. (7) is always equal to zero when \( l \geq D_{\max} \), testing (i) is exactly the same as testing (ii) for \( l \geq D_{\max} \). For \( l < D_{\max} \), testing (i) is special case of testing (ii), i.e., testing (i) is the same as testing (ii) with \( b = 0 \).

By Lemma 6, we guarantee that if (i) is violated for \( l \geq D_{\max} \), (i) is also violated for \( l < D_{\max} \). Since (ii) with \( b = 0 \) is checked, the lemma holds.

So far, we derived an upper-bound of \( l \) to be checked; by Lemma 7, we need to test Eq. (9) only for \( l \leq l'_{\max} \). To further reduce the number of candidates of \( l \) to be checked, we paraphrase Theorem 1 as follows. A task set \( T \) is schedulable by cp-EDF on a uniprocessor platform in the presence of the preemption delay \( \alpha \) if the following condition holds:

\[
\max_{l > 0} \left\{ \frac{\text{LHS of Eq. (9)}}{l} \right\} \leq 1. \tag{10}
\]

In order to utilize the alternative form of Theorem 1 for less number of candidates of \( l \) to be checked, we derive the following lemma.

**Lemma 8**: The LHS of Eq. (10) is maximized when \( l \) or \( l - b \) belongs to \( \Omega \triangleq \{ D_i + n \cdot T_i | \tau_i \in T, n = 0, 1, 2, \cdots \} \).

**Proof**: Suppose that the LHS of Eq. (10) is maximized even though neither \( l \) nor \( l - b \) belongs to \( \Omega \). Let \( l_0 \) and \( b_0 \) denote \( l \) and \( b \) when the LHS of Eq. (10) is maximized.

We show a contradiction.

We consider \( l = l_0 - \epsilon \), where \( \epsilon \) is a sufficiently small value. Since both \( l_0 \) and \( l_0 - b_0 \) do not belong to \( \Omega \), the following inequalities hold for every \( \tau_i \in T : DBF(\tau_i, l_0 - b_0) = DBF(\tau_i, l_0 - b_1), DBF_{\epsilon}(\tau_i, l_0 - b_1) = DBF_{\epsilon}(\tau_i, l_0 - \epsilon - b_0), \) and \( DBF(\tau_i, l_0 - \epsilon) = DBF(\tau_i, l_0 - \epsilon) \). Therefore, the LHS of Eq. (9) for \( l = l_0 - \epsilon \) is the same as that for \( l = l_0 \), but \( l_0 - \epsilon \) itself is smaller than \( l_0 \). This means that (LHS of Eq. (9))/\( l \) for \( l = l_0 - \epsilon \) is larger than that for \( l = l_0 \), which a contradiction.

Then, Lemma 8 indicates that we need to test Eq. (9) only for \( l \) such that \( l \) or \( l - b \) belongs to \( \Omega \). Combining Lemmas 8 and 7 together, we know that the number of candidates of \( l \) (and \( l - b \)) to be checked is \( O(\sum_{\tau \in T} l'_{\max}/T_i) \).

The remaining step is to upper-bound the number of \( b \) to be checked for given \( l \) or \( l - b \). Since we assume a quantum-based time as mentioned in Section 2.1, an upper-bound of the number is \( O(\max_{\tau \in T} C_i) \), which is an upper-bound of \( \beta \) in any case.

Since calculating LHS of Eq. (9) for a given task set with given \( \{X_i\} \) and a given \( l \) and \( b \) requires \( O(n) \), the total time-complexity of testing Theorem 1 for a given task set with given \( \{X_i\} \) is \( O(P) \), where

\[
P = n \cdot \max_{\tau \in T} C_i \cdot \sum_{\tau \in T} l'_{\max}/T_i. \tag{11}
\]

Similar to the fp-EDF analysis without any preemption delay [4] (i.e., Eq. (2) in this paper), the total time-complexity is pseudo-polynomial in the task parameters, if \( U' \) is upper-bounded by a constant that is strictly smaller than 1. Note that the total time-complexity derived here is a rough but safe upper-bound, and we can further reduce the time-complexity by applying a technique to investigate \( l \) more efficiently in [28].