

# Performance Analysis of Distributed Routing Strategies Free of Ping-Pong-Type Looping

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**Abstract**—This paper deals with a distributed adaptive routing strategy which is very simple and effective, and is free of a ping-pong-type looping in the presence of network failures. Using the number of time intervals required for a node to recover from a network failure as the measure of network's adaptability, performance of this strategy and the ARPANET's previous routing strategy (APRS) is comparatively analyzed without resorting to simulation. Formulas of the exact number of time intervals required for failure recovery under both strategies are also derived. We show that i) the performance of the strategy is always better than, or at least as good as, that of APRS, and ii) network topology has significant effects on the performance of both strategies.

We also extend our analytical results to routing strategies which are free of loops with more than two nodes. (Such a loop is called a *multinode loop*.) Finally, numerical examples are presented to demonstrate the utility of our results.

**Index Terms**—ARPANET, distributed adaptive routing strategy, high-order routing strategy, local and global information, multinode loops, network delay table, ping-pong-type loops.

## I. INTRODUCTION

SINCE routing is a key factor to the performance of packet switching networks, various control strategies for routing have been proposed [1]–[5]. Nonadaptive routing strategies, such as random routing and fixed routing, make no attempt to adjust themselves to changes in network conditions and are found to be too inefficient and unreliable to be useful. Centralized adaptive routing strategies—which use a routing control center to keep information on the entire network and make the routing decisions for individual nodes—also suffer from such drawbacks as the vulnerability to single point failures and the need for a routing control center [3]. On the other hand, distributed adaptive routing strategies distribute network information among all computer nodes and, thus, avoid the disadvantages of the other strategies [6]. Hence, we shall focus only on distributed adaptive routing in this paper.

To ensure each node will have sufficient information for routing decisions, we must equip each node with enough information about the entire network. Obviously, it will be very costly for each node to keep and update complete information of the entire network, although, in that case, routing decisions should always be correct. Instead, we would like to incorporate minimal information into each node to make correct or almost correct routing decisions. In other

words, each node keeps only a piece of information (i.e., *local information*) of the network rather than information on the entire network.

Furthermore, the network's adaptability should be taken into account when a distributed routing strategy is utilized. When the local information kept in a node is changed, e.g., some links connected to it become faulty, the node must be able to inform the change to all of its neighbors and let their local information be updated to include this change.

Among all distributed adaptive routing strategies reported in the literature, the ARPANET's previous routing strategy (APRS) appears to be acceptable for most packet switching networks [7]. However, this routing strategy has been replaced by the ARPANET's current routing strategy (ACRS) [8] after being in service for over a decade, due to its poor adaptability to network changes and unnecessary looping in case of network failures, i.e., packets are returned to a node from which they were previously transmitted [8], [9].

The reduction of the looping effects is the subject of this paper. The network's adaptability is improved by incorporating more information in routing messages. Specifically, we shall first consider a routing scheme which turns out to be similar to the one implemented in the TIDAS network [10],<sup>1</sup> and then generalizations thereof. Both our routing strategy and the one in [10] are in principle similar to APRS except that some additional features are included in routing messages to speed up the recovery process in case of a network failure.

Performance analysis of routing strategies is known to be very difficult due to the diversified nature of network structures and is usually carried out via simulation [10]. In this paper, the performance of both ours and APRS is rigorously analyzed without resorting to simulation, and formulas of the exact numbers of time intervals required for failure recovery under both strategies are developed. Also, the effects of network topology on the performance of both strategies are described in theorem form. More importantly, routing strategies free of loops with more than two nodes (called *high-order routing strategies*) are proposed and their performances are analyzed.

The paper is organized as follows. In Section II, we describe both APRS and our strategy whose performance is comparatively analyzed in Section III. In Section IV, we extend our strategy to a more general form which is free of multinode loops. Numerical examples are given to illustrate our analytic results in Section V and the paper concludes with Section VI.

<sup>1</sup> The existence of the strategy in [10] was brought to the authors' attention by an anonymous referee.

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## II. DESCRIPTION OF THE ROUTING STRATEGIES

For a computer network, let  $N_i$ ,  $1 \leq i \leq n$ , be computer nodes and  $L_{i,j}$ ,  $1 \leq i, j \leq n$ , be the communication link from  $N_i$  to  $N_j$  where  $n$  is the total number of nodes in the network. Let  $SP_{i_0, i_m}$  be the set of all paths from  $N_{i_0}$  to  $N_{i_m}$  in the network. A path  $P_i \in SP_{i_0, i_m}$  is expressed by an *ordered* sequence representation  $(N_{i_0}, N_{i_1}, N_{i_2}, \dots, N_{i_m})$ ,  $1 \leq i_j \leq n$  for all  $j \in \{0, 1, 2, \dots, m\}$ , and nodes on the path are visited in that order. The relation  $L_{i_j, i_{j+1}} \in P_i$  means that the link  $L_{i_j, i_{j+1}}$  is a part of the path  $P_i$ . Let  $A_i$  denote the set of all nodes adjacent to  $N_i$ , i.e.,  $\forall N_j \in A_i$ , there exists a communication link  $L_{i,j}$ .

Under APRS, the path from one node to every other node is not determined in advance. Instead, every node maintains a *network delay table* to record the shortest delay via each link emanating from the node. A *minimal delay table* in a node — which contains the delays of the optimal paths (i.e., the path requiring the minimal delay) from that node to all other nodes—is passed to all of its adjacent nodes as a routing message every fixed time interval (i.e., 128 ms in APRS).

Let  $D_{i,j,d}^A(m)$  denote the delay from  $N_i$  via  $N_j$  to  $N_d$  in the network delay table of  $N_i$  under APRS during the time interval  $[m, m+1)$  and  $DL_{i,j}(m)$  be the delay of the link  $L_{i,j}$  at time  $m$ . Also, let  $OP_{j,d}^A(m)$  be the minimum delay path from  $N_j$  to  $N_d$  in the network delay table of  $N_j$  during the interval  $[m, m+1)$  and denote the delay of  $OP_{j,d}^A(m)$  by  $DOP_{j,d}^A(m)$ . According to the operations in APRS, relationships among these quantities can be expressed as follows.

$$D_{i,j,d}^A(m) = DL_{i,j}(m) + DOP_{j,d}^A(m-1) \forall N_j \in A_i \quad (1)$$

$$DOP_{i,d}^A(m) = \min_{N_j \in A_i} \{D_{i,j,d}^A(m)\} \quad (2)$$

On the other hand, under our strategy as well as the one in the TIDAS network, every node keeps a network delay table and exchanges minimal delay tables with all its adjacent nodes as in APRS except for the following modification. If the routing message is passed from  $N_j$  to  $N_i$  which is the second node in the optimal path from  $N_j$  to some destination node  $N_d$ , we replace the delay of the optimal path from  $N_j$  to  $N_d$  with the delay of its *second optimal path* (i.e., the path requiring the second shortest delay to  $N_d$  among all paths in the network delay table of  $N_j$ ) in the routing message passed to  $N_i$ .

For convenience, the APRS and our strategy will henceforth be referred to as strategy A and strategy B, respectively. Let  $SOP_{j,d}^B(m)$ ,  $DSOP_{j,d}^B(m)$ ,  $D_{i,j,d}^B(m)$ ,  $OP_{j,d}^B(m)$  and  $SOP_{j,d}^A(m)$  be defined to have the same meanings as  $SOP_{j,d}^A(m)$ ,  $DSOP_{j,d}^A(m)$ ,  $D_{i,j,d}^A(m)$ ,  $OP_{j,d}^A(m)$  and  $SOP_{j,d}^A(m)$ , respectively, except that they are the notation under strategy B. Also, let  $2nd(P_i)$  denote the second node on the path  $P_i$ . The operations in strategy B can then be expressed in an algorithm form as follows:

For every node  $N_j \in A_i$   
 if  $N_i = 2nd(OP_{j,d}^B(m-1))$  then  $D_{i,j,d}^B(m) = DL_{i,j}(m) + DSOP_{j,d}^B(m-1)$   
 else  $D_{i,j,d}^B(m) = DL_{i,j}(m) + DOP_{j,d}^B(m-1)$

where

$$DOP_{i,d}^B(m) = \min_{N_j \in A_i} \{D_{i,j,d}^B(m)\} \quad (3)$$

$$DSOP_{i,d}^B(m) = \min_{\substack{N_j \in A_i \\ N_j \neq 2nd(OP_{i,d}^B(m))}} \{D_{i,j,d}^B(m)\} \quad (4)$$

Performance of both strategies A and B will be comparatively analyzed first in the following section. In Section IV, the analytic results for strategy B will then be extended to the more general case: routing strategies which are free of multinode loops.

## III. COMPARATIVE ANALYSIS OF NETWORK PERFORMANCE

We begin with an introduction of necessary assumptions and definitions in Section III-A, which is then followed by the performance analysis of strategies A and B in Section III-B. For both strategies the network's adaptability is measured by the number of time intervals required for a node to recover from a network failure.

### A. Assumptions and Definitions

Since node failures can be represented as the failure of all the links attached to it, without loss of generality, only link failures will be considered. Assume that the faulty link  $L_{f,f^*}$  is a part<sup>2</sup> of the optimal path from  $N_s$  to  $N_d$ , and the link failure is detected at time 0 while the delays of other links remain unchanged. We assume further that there always exists at least one nonfaulty path from  $N_s$  to  $N_d$  even after a link failure. In what follows,  $m_A$  and  $m_B$  are used to denote the numbers of time intervals required for  $N_s$  to obtain a new nonfaulty optimal path to  $N_d$  in the presence of a link failure under strategy A and strategy B, respectively.

Also needed are the following definitions.

*Definition 1:* A path  $P_i$  is said to be *faulty* iff the path contains a faulty link  $L_{f,f^*}$ , i.e.,  $L_{f,f^*} \in P_i$ .

*Definition 2:* A *loop*, denoted by  $L_i$ ,  $1 \leq i \leq n$ , is a path which starts and ends at the same node  $N_i$ , i.e.,  $L_i \in SL \equiv \bigcup_{1 \leq i \leq n} SP_{i,i}$ .

*Definition 3:* A path is said to *contain a loop* iff there is one or more nodes appearing more than once in its ordered sequence representation.

*Definition 4:* A path is said to *contain a ping-pong-type loop* iff there exists a node  $N_i$  in its ordered sequence representation which is sandwiched between two occurrences of another node  $N_j$ .

*Definition 5:* If a path  $P_i = (N_{i_0}, N_{i_1}, \dots, N_{i_m})$  is faulty and the pair  $(N_{i_k}, N_{i_{k+1}})$ , i.e.,  $L_{i_k, i_{k+1}}$ , represents the first faulty link in the ordered sequence representation of  $P_i$ , then  $k$  is called the *screen* of  $P_i$  and denoted by  $scn(P_i)$ . If the path  $P_i$  is not faulty, then  $scn(P_i) = \infty$ .

*Definition 6:* A new set of paths,  $SP_{s,d}^*$ , is defined as

$$SP_{s,d}^* \equiv SP_{s,d} - \{P_i | P_i \in SP_{s,d}$$

and  $P_i$  contains a ping-pong-type loop\}.

Obviously,  $SP_{s,d}^* \subseteq SP_{s,d}$ .

<sup>2</sup> If the faulty link is not in the optimal path, both strategies will have the same performance.

**Definition 7:** The *delay function*  $d: SP \rightarrow \mathbf{R}^+$  is defined as sum of delays of all links in a path, where  $SP \equiv \cup_{i \leq i, j \leq n} SP_{i,j}$  and  $\mathbf{R}^+$  is the set of positive real numbers.

**Definition 8:** The *hop function*  $h: SP \rightarrow \mathbf{I}^+$  is defined as the number of links in a path where  $\mathbf{I}^+$  is the set of positive integers.

**Definition 9:** The function  $A: SP \rightarrow \mathbf{R}^+$  represents the *average link delay* in a path, i.e.,  $A(P_i) = [d(P_i)]/[h(P_i)] \forall P_i \in SP$ .

For example, consider a path  $P_i = (N_1, N_2, N_1, N_2, N_1, N_3)$  in the example network of Fig. 1. According to the above definitions,  $d(P_i) = 12$ ,  $h(P_i) = 5$ ,  $A(P_i) = 2.4$ , and  $P_i$  contains a ping-pong-type loop occurring between  $N_1$  and  $N_2$ . Hence,  $P_i \in SP_{1,3}$ , but  $P_i \notin SP_{1,3}^*$ . When the link  $L_{1,3}$  is faulty,  $P_i$  is faulty and  $scn(P_i) = 4$ .

### B. Analysis of the Two Routing Strategies

Under both strategies, each node gathers routing information and then updates its local delay table by exchanging routing messages with all its neighbors every fixed time interval. From (3) and (4), one can see that strategy B removes all the paths with ping-pong-type loops from network delay tables. In case of a link failure, this fact will enable a node under strategy B to determine a new nonfaulty optimal path faster than under strategy A and, thus, enhance the network's adaptability. More formally, this property is stated as a lemma without proof as follows.

**Lemma 1:** Strategy B is free of a ping-pong-type looping, i.e.,  $OP_{s,d}^B(m) \in SP_{s,d}^* \forall m \in \mathbf{I}$  where  $\mathbf{I}$  is the set of integers.

Since the information that the link  $L_{f,f^*} = L_{i_k,i_{k+1}}$  of a path has become faulty is broadcast only one hop per unit time under both strategies, it will take the same number of time units as the screen value of that path for the source node to be informed of the link failure. Thus, the screen values of  $OP_{s,d}^A(m)$  and  $OP_{s,d}^B(m)$  must be greater than  $m$ . This can be formally stated as a lemma below.

**Lemma 2:** The following inequalities hold  $\forall m \in \mathbf{I}^+$ :

$$scn(OP_{s,d}^A(m)) > m$$

$$scn(OP_{s,d}^B(m)) > m.$$

Generally speaking, the screen value of a path means the number of time intervals required to remove this path from the network delay table of  $N_s$  after the occurrence of a failure in the path. Thus, under both strategies, the path with the shortest delay to a destination node  $N_d$  chosen by a source node  $N_s$  is the shortest among all possible paths which have not been found faulty by  $N_s$  up to time  $m$ . More formally, we have the following theorem.

**Theorem 1:**

$$i) DOP_{s,d}^A(m) = \min \{d(P_i) | P_i \in SP_{s,d} \text{ and } scn(P_i) > m\} \forall m \in \mathbf{I}^+,$$

$$ii) DOP_{s,d}^B(m) = \min \{d(P_j) | P_j \in SP_{s,d}^* \text{ and } scn(P_j) > m\} \forall m \in \mathbf{I}^+.$$

**Proof of i):** Recall that  $m_A$  time units are required to find a new nonfaulty optimal path under strategy A in case of a link failure. Consider two cases:  $m \geq m_A$  and  $m < m_A$ . When  $m \geq m_A$ ,  $OP_{s,d}^A(m)$  is the newly found optimal path and fault-

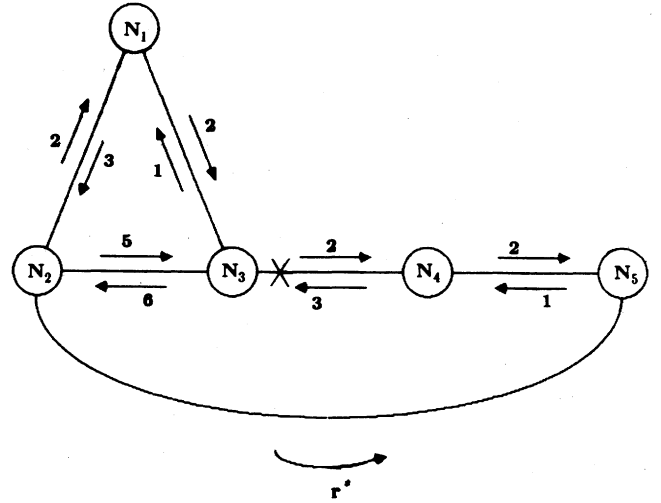


Fig. 1. An example computer network.

free. That is,  $scn(OP_{s,d}^A(m)) = \infty$ , resulting in

$$DOP_{s,d}^A(m) = \min \{d(P_i) | P_i \in SP_{s,d}$$

and  $scn(P_i) = \infty\}$ , thus satisfying i).

When  $m < m_A$ ,  $OP_{s,d}^A(m)$  still contains the faulty link  $L_{f,f^*}$ . We want to show that  $OP_{s,d}^A(m)$  chosen at time  $m$  from the local delay table of  $N_s$  is indeed the real minimum delay path found by  $N_s$ . Let  $P_{i^*}$  be an arbitrary path containing the faulty link  $L_{f,f^*}$  at time  $m$ ,  $P_{i^*} \in SP_{s,d}$  and  $scn(P_{i^*}) > m$ . Then,  $P_{i^*}$  can be expressed as  $(N_{i_0}^*, N_{i_1}^*, \dots, N_{i_r}^*, N_{i_{r+1}}^*, \dots, N_{i_u}^*)$  where  $N_{i_0}^* = N_s$ ,  $N_{i_r}^* = N_f$ ,  $N_{i_{r+1}}^* = N_{f^*}$ ,  $N_{i_u}^* = N_d$ , and  $scn(P_{i^*}) = r > m$ . From (1) and (2) we get

$$DOP_{i_0^*,i_u^*}^A(m) \leq DL_{i_0^*,i_1^*}^A(m) + DOP_{i_1^*,i_u^*}^A(m-1)$$

$$DOP_{i_1^*,i_u^*}^A(m-1) \leq DL_{i_1^*,i_2^*}^A(m-1) + DOP_{i_2^*,i_u^*}^A(m-2)$$

$\vdots$

$$DOP_{i_{r-1}^*,i_u^*}^A(m-r+1) \leq DL_{i_{r-1}^*,i_r^*}^A(m-r+1)$$

$$+ DOP_{i_r^*,i_u^*}^A(m-r)$$

$\Rightarrow$

$$DOP_{i_0^*,i_u^*}^A(m) \leq \sum_{k=0}^{r-1} DL_{i_k^*,i_{k+1}^*}^A(m-k) + DOP_{i_r^*,i_u^*}^A(m-r)$$

$\vdots$

$$\leq \sum_{k=0}^{r-1} DL_{i_k^*,i_{k+1}^*}^A(m-k) + DOP_{i_r^*,i_u^*}^A(m-r).$$

Since  $r > m$ ,  $m$  is a time before  $L_{i_r^*,i_{r+1}^*}$  became faulty and we get

$$DOP_{i_r^*,i_u^*}^A(m-r) = \sum_{k=r}^{u-1} DL_{i_k^*,i_{k+1}^*}^A(m-k)$$

$$\Rightarrow DOP_{s,d}^A(m) = DOP_{i_0^*,i_u^*}^A(m) \leq \sum_{k=0}^{u-1} DL_{i_k^*,i_{k+1}^*}^A(m-k) = d(P_{i^*}).$$

Besides, from Lemma 2,  $\text{scn}(OP_{s,d}^A(m)) > m$  and  $OP_{s,d}^A(m) \in SP_{s,d}$ , thereby leading to i).

*Proof of ii):* This part can be proved similarly to part i) by recursively applying (3) and (4) and retrospectively tracing the network delay tables of nodes in an arbitrary path. ■

The following two important results follow from Theorem 1.

**Result 1:**  $DOP_{s,d}^A(m_A) = DOP_{s,d}^B(m_B)$ . That is, the nonfaulty optimal paths from  $N_s$  to  $N_d$  obtained under both strategies have the same delay.

**Result 2:**  $DOP_{s,d}^A(m) \leq DOP_{s,d}^A(m+1)$  and  $DOP_{s,d}^B(m) \leq DOP_{s,d}^B(m+1) \forall m \in I$ . In other words, the delay of the optimal path chosen under each strategy is a nondecreasing function of time.

Since at least one new nonfaulty path is assumed to exist in case of link failures, after the occurrence of a failure both  $DOP_{s,d}^A(m)$  and  $DOP_{s,d}^B(m)$  will converge to the delay of the new nonfaulty optimal path as  $m$  increases. Let  $r < \infty$  denote the delay of the new nonfaulty optimal path. For a given network,  $m_A$  and  $m_B$  can be determined from the value of  $r$  by the following theorem.

**Theorem 2:**

$$i) m_A = \max \{ \text{scn}(P_i) | P_i \in SP_{s,d} \text{ and } d(P_i) < r \}$$

$$ii) m_B = \max \{ \text{scn}(P_j) | P_j \in SP_{s,d}^* \text{ and } d(P_j) < r \}.$$

*Proof:* Consider i) first. Suppose  $m_A^* = \max \{ \text{scn}(P_i) | P_i \in SP_{s,d} \text{ and } d(P_i) < r \}$  and  $m_A^* \neq m_A$ . We shall prove that both the assumptions of  $m_A > m_A^*$  and  $m_A < m_A^*$  lead to contradictions.

**Case 1:**  $m_A > m_A^*$ . Since  $m_A$  is the minimal number of time intervals required for  $N_s$  to obtain the new nonfaulty optimal path to  $N_d$ ,  $DOP_{s,d}^A(m_A - 1) < DOP_{s,d}^A(m_A) = r$ . By Theorem 1,  $\text{scn}(OP_{s,d}^A(m_A - 1)) = m_A^{**} > m_A - 1 \Rightarrow m_A^{**} \geq m_A$ . Because  $SP_{s,d}$  is the set of all paths from  $N_s$  to  $N_d$ ,

$$m_A^* = \max \{ \text{scn}(P_i) | P_i \in SP_{s,d} \text{ and } d(P_i) < r \} \\ \geq m_A^{**} \geq m_A$$

leading to a contradiction to  $m_A^* < m_A$ .

**Case 2:**  $m_A < m_A^*$ . Since  $m_A^* = \max \{ \text{scn}(P_i) | P_i \in SP_{s,d} \text{ and } d(P_i) < r \}$ , there exists a path  $P_j$  such that  $\text{scn}(P_j) = m_A^*$ ,  $P_j \in SP_{s,d}$  and  $d(P_j) < r$ . From Theorem 1,  $DOP_{s,d}^A(m_A^* - 1) \leq d(P_j) < r$ . For  $m_A, m_A^* \in I^+$ ,  $m_A < m_A^* \Rightarrow m_A \leq m_A^* - 1$ . Thus, we get  $DOP_{s,d}^A(m_A) \leq DOP_{s,d}^A(m_A^* - 1) \leq d(P_j) < r$ . However,  $r = DOP_{s,d}^A(m_A)$ , leading to a contradiction. Since, by Lemma 1,  $OP_{s,d}^B(k) \in SP_{s,d}^* \forall k \in I^+$ , part ii) can be proved similarly to part i). ■

From Theorem 2, we can draw an important conclusion that while some link in the network becomes faulty, the convergence time required for a node to obtain a new nonfaulty optimal path under strategy B is always less than, or equal to, that required under strategy A. We formally state this as a corollary below.

**Corollary 2.1:**  $m_B \leq m_A$ .

*Proof:*  $\forall P_i \in SP_{s,d}^* \Rightarrow P_i \in SP_{s,d}$  since  $SP_{s,d}^* \subseteq SP_{s,d}$ ,  $\Rightarrow m_B \leq m_A$  by Theorem 2. ■

By definition, we can describe the screen and delay functions of a path  $P_i = (N_{i_0}, N_{i_1}, \dots, N_{i_k}, N_{i_{k+1}}, \dots, N_{i_u})$

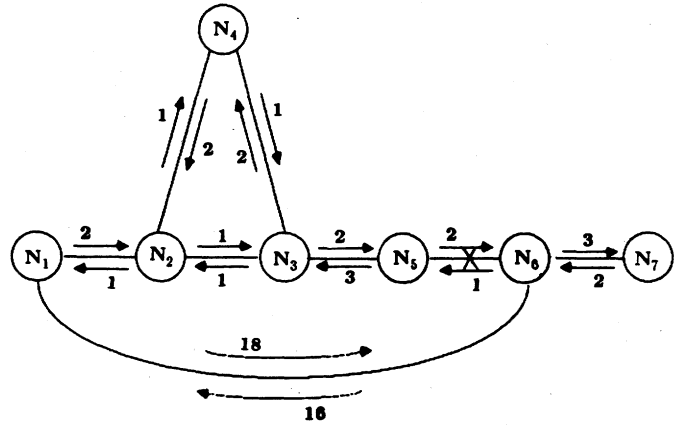


Fig. 2. An example network showing  $r = 21$  when  $L_{5,6}$  becomes faulty.

$\in SP_{s,d}$  where  $N_{i_0} = N_s$ ,  $N_{i_k} = N_f$ ,  $N_{i_{k+1}} = N_{f^*}$ , and  $N_{i_u} = N_d$  as below.

$$\text{scn}(P_i) = h(P_i') + \sum_{j=1}^k n_{ij} h(L_{ij}) \quad (5)$$

$$d(P_i) = d(P_i') + \sum_{j=1}^k n_{ij} d(L_{ij}) + d(P_{i_r}) \quad (6)$$

where  $P_i \supset P_{i_r} \in SP_{f,d}$ ,  $P_i' \in SP_{s,f}$ ,  $L_{ij}$  is a loop starting and ending at  $N_{i_j}$  and  $n_{ij}$  the number of times the loop  $L_{ij}$  appears in the path  $P_i$ .

Since we can include all redundant parts of  $P_i$  in the second term of (5) and the second and third terms of (6),  $P_i'$  can be regarded as a path without loops. For the example network of Fig. 2, consider a path  $P_i = (N_1, N_2, N_3, N_4, N_2, N_3, N_5, N_3, N_5, N_3, N_5, N_6, N_7)$  in which the link  $L_{5,6}$  is faulty. Then  $P_i' = (N_1, N_2, N_3, N_5)$ ,  $P_{i_r} = (N_5, N_6, N_7)$ ,  $L_{i_1} = (N_2, N_3, N_4, N_2)$ ,  $n_{i_1} = 1$  and  $L_{i_2} = (N_3, N_5, N_3)$ ,  $n_{i_2} = 2$ . Using (5) and (6), we can express Theorem 2 as the following optimization problems.

$$m_A: \max_{P_i \in SP_{s,d}} h(P_i') + \sum_{j=1}^k n_{ij} h(L_{ij}) \quad (7)$$

$$\text{subject to } d(P_i') + \sum_{j=1}^k n_{ij} d(L_{ij}) + d(P_{i_r}) \\ < r \text{ where } L_{ij} \in SP_{i_j, i_j} \text{ and } L_{ij} \subset P_i. \quad (7')$$

$$m_B: \max_{P_i \in SP_{s,d}^*} h(P_i') + \sum_{j=1}^k n_{ij} h(L_{ij}) \quad (8)$$

$$\text{subject to } d(P_i') + \sum_{j=1}^k n_{ij} d(L_{ij}) + d(P_{i_r}) \\ < r \text{ where } L_{ij} \in SP_{i_j, i_j}^* \text{ and } L_{ij} \subset P_i. \quad (8')$$

It is interesting to observe that the  $n_{ij}$ 's satisfying (7) and (7') (or (8) and (8')) can be determined by solving a linear knapsack problem. For example, consider the determination of  $m_A$ . Suppose  $P_{i^*} \in SP_{s,d}$  is the path which maximizes the expression of (7) subject to the constraint (7'). The knowledge of the path  $P_{i^*}$  completely specifies  $P_{i^*}'$ ,  $P_{i_r}^*$ ,  $L_{ij}^*$ , and  $n_{ij}^*$  for

$1 \leq j \leq k$ . Since  $SP_{s,d}$  is the set of all paths from  $N_s$  to  $N_d$ , from (7) and (7') the vector  $\mathbf{n} = [n_1^*, n_2^*, \dots, n_k^*]$  is the solution to a linear integer knapsack programming problem  $P_0$ , which can be obtained by applying the algorithm  $A_0$  given below.

$$P_0: S = \max_{n_i, 1 \leq i \leq k} \sum_{i=1}^k a_i n_i$$

subject to  $\sum_{i=1}^k b_i n_i < M$  and  $n_i \in I^+$

for  $i=1, \dots, k$

where  $S = m_A - h(P_{i^*}^*)$ ,  $h(L_{j^*}^*) = a_i$ ,  $d(L_{j^*}^*) = b_i$  and  $M = r - d(P_{i^*}^*) - d(P_{i^*}^*)$ .

**Algorithm  $A_0$ :**

*Step 1)* Set  $m := M$  and  $n_i := 0$  for all  $i \in \{1, \dots, k\}$ .

*Step 2)* Select an  $i^*$  such that  $a_{i^*}/b_{i^*} = \max_{b_i < m} \{a_i/b_i\}$ .

*Step 3)*  $n_{i^*} := \lfloor m/b_{i^*} \rfloor$ ,  $m := m - b_{i^*} n_{i^*}$ .

*Step 4)* If there exists a  $b_i < m$  then go to Step 2) else stop.

The determination of  $m_B$  can be treated similarly and the above result leads to the following lemma.

**Lemma 3:**  $S = \lfloor M/b_{i^*} \rfloor a_{i^*} + R$  where  $R < a_{i^*}$  and  $a_{i^*}/b_{i^*} = \max_{1 \leq i \leq k} \{a_i/b_i\}$ .

*Proof:* Let  $\mathbf{n}^* = [n_1^*, n_2^*, \dots, n_k^*]$  be a solution of  $P_0$  obtained by applying algorithm  $A_0$ .

$$S = \sum_{i=1}^k a_i n_i^* = \left\lfloor \frac{M}{b_{i^*}} \right\rfloor a_{i^*} + \sum_{i \neq i^*}^k a_i n_i^*$$

$$\cdot \sum_{i=1}^k b_i n_i^* < M \Rightarrow \sum_{i \neq i^*}^k b_i n_i^* < M - \left\lfloor \frac{M}{b_{i^*}} \right\rfloor b_{i^*} < b_{i^*}$$

$$\Rightarrow \sum_{i \neq i^*}^k a_i n_i^* < a_{i^*} \quad \text{because} \quad \frac{a_{i^*}}{b_{i^*}} = \max_{1 \leq i \leq k} \left\{ \frac{a_i}{b_i} \right\}.$$

Thus, Lemma 3 follows.  $\blacksquare$

Define loops  $L_A$  and  $L_B$  such that  $A(L_A) = \min \{A(L_i) | L_i \in SP_{i,i}, 1 \leq i \leq n\}$ , and  $A(L_B) = \min \{A(L_i) | L_i \in SP_{i,i}^*, 1 \leq i \leq n\}$ . From (7) and Lemma 3, when the delay of the new nonfaulty optimal path  $r$  is large enough,  $A(L_A)$  will become the dominating factor in determining  $m_A$ . Clearly, what  $L_A$  is to  $m_A$  is  $L_B$  to  $m_B$ . This fact leads to Theorem 3 below.

**Theorem 3:**

$$\lim_{r \rightarrow \infty} \frac{m_B}{m_A} = \frac{A(L_A)}{A(L_B)}.$$

*Proof:* Let  $L_{i_m}$  be the loop with the minimal average delay among all loops in the path  $P_i$ , i.e.,  $A(L_{i_m}) = \min \{A(L_j) | L_j \in SL \text{ and } L_j \subseteq P_i\}$ . Applying Lemma 3 to (7) and (8), we get

$$m_A = \max_{P_i \in SP_{s,d}} \left\{ \left[ \frac{r - d(P_i') - d(P_i)}{d(L_{i_m})} \right] \cdot h(L_{i_m}) + R_i + h(P_i') \right\} \quad \text{where } R_i < h(L_{i_m}). \quad (9)$$

$$m_B = \max_{P_i \in SP_{s,d}^*} \left\{ \left[ \frac{r - d(P_i') - d(P_i)}{d(L_{i_m})} \right] \cdot h(L_{i_m}) + R_i + h(P_i') \right\} \quad \text{where } R_i < h(L_{i_m}). \quad (10)$$

Clearly, the second and third terms in these two equations become negligible as  $r \rightarrow \infty$ . Therefore

$$\lim_{r \rightarrow \infty} \frac{m_B}{m_A} = \frac{d(L_A)h(L_B)}{d(L_B)h(L_A)} = \frac{A(L_A)}{A(L_B)}. \quad \blacksquare$$

The first and second terms in (9) and (10) are related to the redundant parts of a path, while the third term can be viewed as the distance (measured in hops) to the faulty link. Theorem 3 shows that the improvement via strategy B strongly depends on network topology. This fact will be illustrated with the examples in Section V.

#### IV. ROUTING STRATEGIES WITHOUT MULTINODE LOOPS

Although ping-pong-type loops can be removed by strategy B, loops with more than two nodes (i.e., multinode loops) may still exist under both strategies. Multinode loops, if they occur, usually cause much severer degradation in network performance. Consequently, it is important to develop a routing strategy which eliminates multinode loops.

Strategy B removes ping-pong-type loops by disallowing each node to send its neighbors the routing messages which have been found to be useless to receivers. However, under strategy B every node can determine only the delay and the second node of each path from its network delay table. Naturally, as the amount of local information is increased, every node is expected to form more useful routing messages leading to a greater reduction of looping effects. In fact, a routing strategy can be developed to eliminate multinode loops by keeping in network delay tables not only the delay of each path but also the set of nodes which follow the first node in the ordered sequence representation of the path. Thus, the format of routing messages is modified to include the corresponding set of nodes in each path. The augmented information in the network delay table of each node will be very useful in forming suitable routing messages for its neighbors.

The number of nodes included in the routing message will henceforth be referred to as the *order* of the routing strategy. As the order of the routing strategy gets higher, it will become free of higher order loops, i.e., loops with more number of nodes. Note that strategies A and B are actually the routing strategies whose orders are 0 and 1, respectively. (A simple example for the operations under the second order routing strategy is presented in the next section.)

Performance of the  $k$ th-order routing strategy can be analyzed using the same approach as in Section III. The following definition is required to facilitate our discussion.

**Definition 10:** A path  $P_i$  is said to be *free of  $k$ th-order loops* iff for any two occurrences of a node in its ordered sequence representation there are at least  $k + 1$  nodes between them.

Let  $SP^k$ ,  $k \in I^+$ , be the set of paths which are free of  $k$ th

order loops and  $SP_{i,j}^k \equiv SP^k \cap SP_{i,j}$  be the set of paths from  $N_i$  to  $N_j$  and free of  $k$ th-order loops. Obviously,  $SP^{k+1} \subseteq SP^k, \forall k \in I^+$ . Since the relationship of  $SP_{s,d}^*$  to strategy B is the same as the relationship of  $SP_{s,d}^k$  to the  $k$ th-order routing strategy, the node  $N_s$  under the  $k$ th-order routing strategy chooses its nonfaulty optimal paths only from the set  $SP_{s,d}^k$ . The entries of the network delay table during failure recovery can be clearly described by the following theorem, whose proof is similar to that of Theorem 1 and, thus, omitted.

*Theorem 4:*

$$DOP_{s,d}^k(m) = \min \{d(P_j) | P_j \in SP_{s,d}^k, \text{ and } \text{scn}(P_j) > m\} \quad \forall m \in I^+.$$

It can be verified by Theorem 4 that the delays of nonfaulty optimal paths from  $N_s$  to  $N_d$  obtained under all strategies are indeed the same, i.e.,  $DOP_{s,d}^i(m_i) = DOP_{s,d}^j(m_j) = r, \forall i, j \in I^+$  where  $m_i$  is the number of time intervals for  $N_s$  to determine a new nonfaulty optimal path to  $N_d$  under the  $i$ th-order routing strategy after the occurrence of a link failure. From Theorem 4, we have the following theorem to determine  $m_k \forall k \in I^+$ , which is an extended version of Theorem 2 and can be proven similarly.

*Theorem 5:*

$$m_k = \max \{ \text{scn}(P_j) | P_j \in SP_{s,d}^k \text{ and } d(P_j) < r \}.$$

From this theorem, we get  $m_{k+1} \leq m_k \forall k \in I^+$ . This implies that the network's adaptability is improved monotonically by adding more information in routing messages. Moreover, with the same reasoning as in Section III we can obtain a generalized form of (7) and (7') as follows:

$$m_k: \max_{P_i \in SP_{s,d}^k} h(P_i') + \sum_{j=1}^k n_{ij} h(L_{ij}) \quad (11)$$

subject to  $d(P_i') + \sum_{j=1}^k n_{ij} d(L_{ij}) + d(P_{ir})$

$$< r \text{ where } L_{ij} \in SP_{ij,i,j}^k \text{ and } L_{ij} \subset P_i \quad (11')$$

where  $P_i', P_{ir}, n_{ij}$  and  $L_{ij}$  are defined as before.

Define  $L_*^k$  to be a loop such that  $A(L_*^k) = \min \{A(L_i) | L_i \in SP_{i,i}^k, 1 \leq i \leq n\}$ . When the delay of the new nonfaulty optimal path  $r$  is large enough,  $A(L_*^k)$  will become the dominating factor in determining  $m_k$  and we have the following extended results for high-order routing strategies.

*Theorem 6:*

$$\lim_{r \rightarrow \infty} \frac{m_i}{m_j} = \frac{A(L_*^i)}{A(L_*^j)}, \quad \forall i, j \in I^+.$$

*Proof:* Let  $L_{i_m}$  be the loop with the minimal average delay among all loops in the path  $P_i$ . Then, from (11) we have

$$m_k = \max_{P_i \in SP_{s,d}^k} \left\{ \left[ \frac{r - d(P_i') - d(P_{ir})}{d(L_{i_m})} \right] h(L_{i_m}) + R_i + h(P_i') \right\} \quad \text{where } R_i < h(L_{i_m}). \quad (12)$$

Following a proof similar to the proof of Theorem 3, the desired result is obtained. ■

The above results indicate that the network's adaptability is actually enhanced with the increased amount of local information. However, according to experimental results in [5], multinode loops occur very infrequently. Besides, the operational overheads required in higher order routing strategies cannot be ignored and the feasibility of implementing higher order routing strategies must be justified. To this end, Theorem 5 provides a good indication for the usefulness of a high-order routing strategy in a certain network and can also be used to determine the minimal order of routing strategy required in a given network to remove all possible looping.

## V. EXAMPLES

In this section, two examples are presented to illustrate our analytic results obtained thus far.

*Example 1:* In a computer network shown in Fig. 1, assume that the link  $L_{3,4}$  became faulty at time 0 and delays of the other links remain constant. From (9) and (10), the number of time intervals required for each node in this network to obtain its new nonfaulty optimal path is a function of  $r^* = L_{2,5}$ . Thus, network performance has been examined while this parameter is being varied.

*Case 1:  $r^* = 16$*

Given below are the operations taken under strategies A and B for the source node  $N_1$  to obtain a new nonfaulty optimal path to the destination node  $N_5$  by exchanging routing messages with neighboring nodes every time interval.

Under strategy A, network delay tables become as shown in Table I. From Table I, one can obtain

$$OP_{1,5}^A(t_0) = (N_1, N_3, N_4, N_5) \quad \text{where } t_0 \in (-\infty, 0)$$

$$OP_{1,5}^A(0) = (N_1, N_3, N_4, N_5)$$

$$OP_{1,5}^A(1) = OP_{1,5}^A(2) = (N_1, N_3, N_1, N_3, N_4, N_5)$$

$$OP_{1,5}^A(3) = OP_{1,5}^A(4) = (N_1, N_3, N_1, N_3, N_1, N_3, N_4, N_5)$$

$$OP_{1,5}^A(5) = OP_{1,5}^A(6) = (N_1, N_3, N_1, N_3, N_1, N_3, N_1, N_3, N_4, N_5)$$

$$OP_{1,5}^A(7) = OP_{1,5}^A(8) = (N_1, N_3, N_1, N_3, N_1, N_3, N_1, N_3, N_1, N_3, N_4, N_5)$$

$$OP_{1,5}^A(9) = (N_1, N_2, N_5).$$

Thus,  $m_A = 9$ , while  $\text{scn}(OP_{1,5}^A(8)) = 9$ . This agrees with the result of Theorem 2. In addition, one can observe from the above tables that the numbers of time intervals for  $N_2$  and  $N_3$  to obtain the new nonfaulty optimal paths are 6 and 10, respectively (marked by \*).

On the other hand, we obtain the network delay tables under

TABLE I  
NETWORK DELAY TABLES OF  $N_1N_2$  AND  $N_3$  UNDER STRATEGY A WHERE  $N_5$  IS THE DESTINATION NODE (a) NETWORK DELAY TABLE OF  $N_1$  (b) NETWORK DELAY TABLE OF  $N_2$  (c) NETWORK DELAY TABLE OF  $N_3$

Entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$	$t \in [11, \infty)$
$N_2$	11	11	11	11	14	14	17	17	19	19	19*	19	19
$N_3$	6	6	9	9	12	12	15	15	18	18	21	21	22

(a)

Entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t=10$	$t \in [11, \infty)$
$N_1$	8	8	8	11	11	14	14	17	17	20	20	21	21
$N_3$	9	9	12	12	15	15	18	18	21	21	24	24	25
$N_5$	16	16	16	16	16	16	16	16*	16	16	16	16	16

(b)

Entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$	$t \in [10, \infty)$
$N_1$	7	7	7	10	10	13	13	16	16	19	19	20*
$N_2$	14	14	14	14	17	17	20	20	22	22	22	22
$N_4$	4	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$

(c)

TABLE II  
NETWORK DELAY TABLES OF  $N_1N_2$  AND  $N_3$  UNDER STRATEGY B WHERE  $N_5$  IS THE DESTINATION NODE (a) NETWORK DELAY TABLE OF  $N_1$  (b) NETWORK DELAY TABLE OF  $N_2$  (c) NETWORK DELAY TABLE OF  $N_3$

Entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t \in [4, \infty)$
$N_2$	12	12	12	19	19	19*
$N_3$	6	6	16	16	16	24

(a)

Entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t \in [5, \infty)$
$N_1$	8	8	8	18	18	18	26
$N_3$	9	9	18	18	18	25	25
$N_5$	16	16	16	16*	16	16	16

(b)

Entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t \in [3, \infty)$
$N_1$	13	13	13	13	20*
$N_2$	14	14	14	14	22
$N_4$	4	$\infty$	$\infty$	$\infty$	$\infty$

(c)

strategy B as shown in Table II. From this table we get

$$OP_{1,5}^B(t_0) = (N_1, N_3, N_4, N_5) \text{ where } t_0 \in (-\infty, 0)$$

$$OP_{1,5}^B(0) = (N_1, N_3, N_4, N_5)$$

$$OP_{1,5}^B(1) = (N_1, N_2, N_3, N_4, N_5)$$

$$OP_{1,5}^B(2) = OP_{1,5}^B(3) = (N_1, N_3, N_2, N_1, N_3, N_4, N_5)$$

$$OP_{1,5}^B(4) = (N_1, N_2, N_5)$$

$$m_B = 4, \text{scn}(OP_{1,5}^B(3)) = 4.$$

The numbers of time intervals required for  $N_2$  and  $N_3$  to obtain the new optimal paths are also reduced from six and ten to two and three, respectively.

Case 2:  $r^* = 160$

Applying the same procedure as above, we can obtain the

results for  $r^* = 160$  with a simple calculation. The numbers of time intervals required for  $N_1, N_2,$  and  $N_3$  to obtain their new optimal paths to  $N_5$  are reduced from 105, 102, and 106 to 50, 49, and 51, respectively.

Case 3:  $r^* = 1000$

In this case, the numbers of time intervals for  $N_1, N_2,$  and  $N_3$  to obtain their optimal paths to  $N_5$  are 665, 662, and 666 under strategy A and 332, 331, and 333 under strategy B. Clearly, the performance improvement is around 50 percent.

In Fig. 1,  $L_A = (N_1, N_3, N_1)$  and  $L_B = (N_1, N_2, N_3, N_1)$ . Since  $d(L_A) = 3, h(L_A) = 2, d(L_B) = 9$  and  $h(L_B) = 3, A(L_A) = 1.5$  and  $A(L_B) = 3$ . The results of Cases 2 and 3 agree with Theorem 3.

Example 2: Consider the example network shown in Fig. 3. Applying the same procedure as in Example 1, the time intervals required for  $N_1, N_2, N_3,$  and  $N_4$  to determine their new optimal paths to  $N_5$  become 20, 19, 17, and 20, respectively, under strategy A (i.e., the 0th-order routing strategy), and 11, 10, 8, and 9, respectively, under strategy B (i.e., the first-order routing strategy). Table III illustrates the operations under the 2nd-order routing strategy. The subscript of each entry in network delay tables represents the set of the second and the third nodes of the corresponding path. The result shows that the required time intervals become 6, 5, 5, and 4, respectively. It is easy to see that this is a significant improvement over lower order (i.e., 0th and first) routing strategies.

With enough routing information, a node can find some of its neighbors unable to offer any loop-free path, and thus remove them from consideration. The entries in Table III marked by ~'s represent such cases. While the network in Fig. 3 is still subject to looping under the 2nd-order routing strategy, it can be easily verified that the 2nd-order routing strategy will remove all possible looping in the network in Fig. 1 regardless of the value of  $r^*$ . This fact implies that the required order of the routing strategy to remove all looping depends on the network structure.

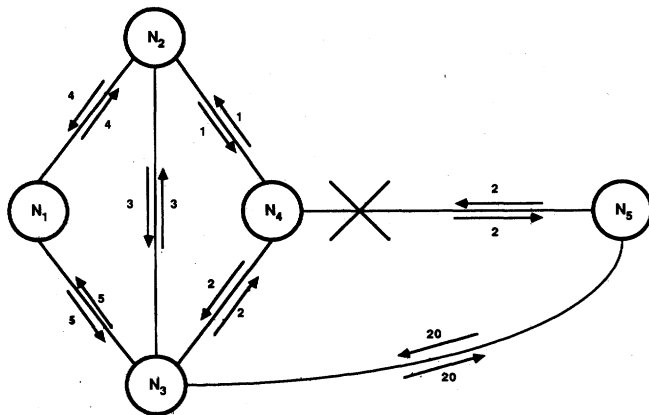


Fig. 3. An example network with link delays specified.

TABLE III

NETWORK DELAY TABLES OF  $N_1, N_2, N_3$  AND  $N_4$  UNDER THE SECOND-ORDER ROUTING STRATEGY WHERE  $N_5$  IS THE DESTINATION NODE (a) NETWORK DELAY TABLE OF  $N_1$  (b) NETWORK DELAY TABLE OF  $N_2$  (c) NETWORK DELAY TABLE OF  $N_3$  (d) NETWORK DELAY TABLE OF  $N_4$ .

entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t \in [6, \infty)$
$N_2$	$7_{(2,4)}$	$7_{(2,4)}$	$7_{(2,4)}$	$11_{(2,3)}$	$19_{(2,4)}$	$19_{(2,4)}$	$19_{(2,4)}$	$27_{(2,4)}$
$N_3$	$9_{(3,4)}$	$9_{(3,4)}$	$9_{(3,4)}$	$11_{(3,2)}$	$21_{(3,4)}$	$21_{(3,4)}$	$21_{(3,4)}$	$25_{(3,5)^*}$

(a)

entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t \in [7, \infty)$
$N_1$	$13_{(1,2)}$	$13_{(1,2)}$	$13_{(1,2)}$	$13_{(1,2)}$	$\sim$	$25_{(1,3)}$	$25_{(1,2)}$	$25_{(1,2)}$	$29_{(1,2)}$
$N_3$	$7_{(3,4)}$	$7_{(3,4)}$	$7_{(3,4)}$	$23_{(3,4)}$	$23_{(3,4)}$	$23_{(3,4)}$	$23_{(3,4)^*}$	$23_{(3,4)}$	$23_{(3,4)}$
$N_4$	$3_{(4,4)}$	$3_{(4,4)}$	$15_{(4,3)}$	$15_{(4,3)}$	$15_{(4,3)}$	$15_{(4,3)}$	$23_{(4,3)^*}$	$23_{(4,3)}$	$23_{(4,3)}$

(b)

entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t \in [7, \infty)$
$N_1$	$12_{(1,2)}$	$12_{(1,2)}$	$12_{(1,2)}$	$12_{(1,2)}$	$\sim$	$24_{(1,2)}$	$24_{(1,2)}$	$24_{(1,2)}$	$32_{(1,2)}$
$N_2$	$6_{(2,4)}$	$6_{(2,4)}$	$6_{(2,4)}$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$	$\sim$
$N_4$	$4_{(4,4)}$	$4_{(4,4)}$	$16_{(4,2)}$	$16_{(4,2)}$	$16_{(4,2)}$	$16_{(4,2)}$	$\sim$	$\sim$	$\sim$
$N_5$	$20_{(4)}$	$20_{(4)}$	$20_{(4)}$	$20_{(4)}$	$20_{(4)}$	$20_{(4)}$	$20_{(4)^*}$	$20_{(4)}$	$20_{(4)}$

(c)

entry	$t_0 \in (-\infty, 0)$	$t=0$	$t=1$	$t=2$	$t=3$	$t \in [4, \infty)$
$N_2$	$14_{(2,1)}$	$14_{(2,1)}$	$14_{(2,1)}$	$14_{(2,1)}$	$14_{(2,1)}$	$24_{(2,3)}$
$N_3$	$14_{(3,1)}$	$14_{(3,1)}$	$14_{(3,1)}$	$14_{(3,1)}$	$14_{(3,1)}$	$22_{(3,5)^*}$
$N_5$	$2_{(5)}$	$\infty$	$\infty$	$\infty$	$\infty$	$\infty$

(d)

VI. CONCLUSION

In this paper, we have rigorously analyzed the performance of two distributed adaptive routing strategies without resorting to simulation. We have also derived formulas for the number of time intervals required for a node to determine its new nonfaulty optimal path to any other destination node in the presence of link/node failures in the network. These results are then generalized to high-order routing strategies which are free of multinode loops. Moreover, for a given network we can determine the importance of each link/node by estimating the average number of time intervals required to recover from the failure of that link/node. That is, the link/node requiring a longer average convergence time for failure recovery possesses more importance.

Note that the order of a routing strategy determines the maximum number of nodes in an extant loop under that strategy. If we adopt a higher order routing strategy, the looping effects can be reduced further but it will induce higher operational overheads in processing the associated routing

messages. It will require a complex procedure to optimally (in some sense) strike a compromise between these two mutually conflicting factors. This is a matter of our future research.

APPENDIX

LIST OF SYMBOLS

- $SP$  Set of all paths in the network.
- $SP_{s,d}$  Set of all paths from  $N_s$  to  $N_d$ .
- $SP_{s,d}^*$  Set of all paths from  $N_s$  to  $N_d$ , which are free of ping-pong-type loops.
- $SP_{s,d}^k$  Set of all paths from  $N_s$  to  $N_d$ , which are free of  $k$ th-order loops.
- $A_i$  Set of all nodes adjacent to  $N_i$ .
- $DL_{i,j}(m)$  The delay associated with  $L_{i,j}$  at time  $m$ .
- $D_{i \setminus j,d}^A(m)$  The delay from  $N_i$  via  $N_j$  to  $N_d$  in the network delay table of  $N_i$  under strategy A during the time interval  $[m, m + 1)$ .
- $OP_{s,d}^A(m)$  Path with the shortest delay from  $N_s$  to  $N_d$  in the network delay table of  $N_s$  under strategy A during the time interval  $[m, m + 1)$ .
- $DOP_{s,d}^A(m)$  The delay of  $OP_{s,d}^A(m)$ .
- $D_{i \setminus j,d}^B(m)$  The delay from  $N_i$  via  $N_j$  to  $N_d$  in the network delay table of  $N_i$  under strategy B during the time interval  $[m, m + 1)$ .
- $OP_{s,d}^B(m)$  Path with the shortest delay from  $N_s$  to  $N_d$  in the network delay table of  $N_s$  under strategy B during the time interval  $[m, m + 1)$ .
- $DOP_{s,d}^B(m)$  The delay of  $OP_{s,d}^B(m)$ .
- $SOP_{s,d}^B(m)$  Path with the second shortest delay from  $N_s$  to  $N_d$  in the network delay table of  $N_s$  under strategy B during the time interval  $[m, m + 1)$ .
- $DSOP_{s,d}^B(m)$  The delay of  $SOP_{s,d}^B(m)$ .
- $OP_{s,d}^k(m)$  Path with the shortest delay from  $N_s$  to  $N_d$  in the network delay table of  $N_s$  under the  $k$ th-order routing strategy during the time interval  $[m, m + 1)$ .
- $DOP_{s,d}^k(m)$  The delay of  $OP_{s,d}^k(m)$ .
- $m_A$  The number of time units required for  $N_s$  to obtain its new nonfaulty optimal path to  $N_d$  in case of a link failure under strategy A.
- $m_B$  The number of time units required for  $N_s$  to obtain its new nonfaulty optimal path to  $N_d$  in case of a link failure under strategy B.
- $m_k$  The number of time units required for  $N_s$  to obtain its new nonfaulty optimal path to  $N_d$  in case of a link failure under the  $k$ th-order routing strategy.

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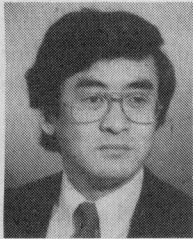
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The Channel Access Protocol (CAP) is a carrier-sense multiple access protocol with staggered delays (CSMA/CD) [1]. [2]. The CSMA/CD protocol provides each adapter with channel access under extremely low probability of collision. In which case the protocol also provides a collision resolution mechanism to principle, each adapter schedules its transmission at a different time. These times are offset so that the distance to ongoing, earlier initiated transmissions can be observed by every adapter. A sending adapter will thus initiate a transmission only when the channel is sensed idle, ensuring collision free transmission of its message. To implement this scheduling mechanism, adapters are synchronized, except during periods of silence, and are

VERY high-speed networks support data rates of several tens of Mbit/s and typically connect a small number of devices over a short distance [8]. High-speed networks are generally used to interconnect high-speed heterogeneous devices such as main frames [4]-[7]. Such interconnection can support distributed computation, resource sharing, etc. From hardware design and performance considerations data rate increase is best provided by the introduction of additional channels [1]-[4], [9], [10], [12].

HYPERchannel is a commercially available and widely used very high-speed network [1]-[3], [8], [11]. In the "HYPERchannel" network up to four channels or "trunks" can be used in parallel. These are connected to each station via a bus interface unit (BIU) wired adapter. The adapter provides the station's access mechanism to the channel and the channel selection mechanism through which a specific channel can be chosen for transmission or reception of messages.

Only a limited number of studies are available for performance evaluation of HYPERchannel networks [8], [9], [11]. They concentrate on the analysis and measurement of the

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