COORDINATION OF DUAL ROBOT ARMS USING KINEMATIC REDUNDANCY *

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ABSTRACT

A new method is developed to coordinate the motion of dual robot arms carrying a solid object, where the first robot (leader) grasps one end of the object rigidly and the second robot (follower) is allowed to change its grasping position at the other end of the object along the object surface while supporting the object. It is shown that this flexible grasping is equivalent to the addition of one more degree of freedom (dof), giving the follower more maneuvering capabilities. Especially, motion commands for the follower are generated by using kinematic redundancy.

To show the utility and power of the method, an example system with two PUMA 560 robots carrying a beam is analyzed in detail.

1 INTRODUCTION

Recently, several methods have been proposed to coordinate the control of two robot arms carrying a solid object [1] [2] [3] [4], where the common solid object was assumed to be rigidly grasped by both arms. The relative position and orientation of the two robot arms were fixed during the entire execution of the robots' task. Under this setting, however, some tasks such as carrying an object along a prespecified path with two robots may not be accomplished due to the insufficient number of degrees of freedom (dofs) available to them. Call one of the two robots the *leader* and the other the *follower*. Consider the motion planning for these two robots, where motion commands for the follower are to be generated based on the leader's motion. When the leader's motion leads the follower to a singular region, or when the motion command generated from the leader's motion requires the follower to violate its joint limits and/or to collide with obstacles, the task cannot be accomplished with the invariant grasping position unless the desired path is modified or the number of the follower's dofs is increased.

In this paper, a new method to overcome the above difficulty is proposed by relaxing the assumption of invariant grasping position of the follower. With this relaxation, the follower will first be shown to be considered as a redundant manipulator without physically adding joints to it. Then, the follower's motion commands are generated by employing the kinematic control techniques commonly used for redundant manipulators [5] [6] [7] [8]. The kinematic control of a redundant manipulator is known to find joint angles and/or velocities such that its end effector attains desired positions and orientations while minimizing some cost function. The object surface is required to be smooth enough for the follower to change its grasping position while keeping its (supporting) orientation and position in order not to drop the object. This requirement may be satisfied by placing ball bearings on the follower's palm such that the sliding contact friction between the follower and the object can be made sufficiently small for the follower's hand to move along the object surface smoothly. It is remarked that our method may be considered as a generalization of the Zheng and Luh's method [1] in the sense that if the smoothness assumption of the object does not hold, then our method reduces to theirs.

The organization of the paper is as follows. In Section 2, a problem formulation is presented along with the definition of "supporting orientation". In Section 3, a solution approach is proposed by employing the control techniques for redundant manipulators. In Section 4, a numerical example is presented to show the utility and power of our proposed method, where two PUMA 560 manipulators are considered. The paper concludes with Section 5.

2 PROBLEM FORMULATION

Consider two robots each with n joints carrying a rigid object as shown in Fig. 1, which is too large and too long for a single robot to handle. The task given to the two robots is to move the object from one location to another along a prescribed path, while not exceeding joint limits and/or avoiding obstacles. Under the smoothness assumption mentioned earlier, the problem is to determine the joint trajectories of two robots as well as the trajectory of the follower's gripping position to accomplish the task.

Let $(x^o,\ y^o,\ z^o)$, $(x_n^l,\ y_n^l,\ z_n^l)$, and $(x_n^f,\ y_n^f,\ z_n^f)$ be the coordinate frames of object, end effectors of the leader and the follower, respectively (see Fig. 1). Let $\mathbf{r}_1$ and $\mathbf{r}_2$ be the position vectors of the origins of $(x^o,\ y^o,\ z^o)$ and $(x_n^f,\ y_n^f,\ z_n^f)$ with reference to $(x_n^l,\ y_n^l,\ z_n^l)$ and $(x^o,\ y^o,\ z^o)$, respectively. Let $\mathbf{H}_1$ and $\mathbf{H}_2$ be the 4×4 homogeneous matrices representing the coordinate frame of the object with reference

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to $(x_n^l,\ y_n^l,\ z_n^l)$ and that of the follower's end effector with reference to $(x^o,\ y^o,\ z^o)$, respectively. Also, let $\mathbf{q}$ be an n-dimensional vector representing joint positions, and $\mathbf{n}(\mathbf{q})$, $\mathbf{s}(\mathbf{q})$, $\mathbf{a}(\mathbf{q})$ and $\mathbf{p}(\mathbf{q})$ be the normal, sliding, approaching and position vectors of the leader with reference to the base coordinate $(x_B,\ y_B,\ z_B)$. Define $\mathbf{T}_B^n(\mathbf{q})$ and $\mathbf{R}_B^n(\mathbf{q})$ as

$$\mathbf{T}_B^n(\mathbf{q}) \triangleq \begin{bmatrix} \mathbf{n}(\mathbf{q}) & \mathbf{s}(\mathbf{q}) & \mathbf{a}(\mathbf{q}) & \mathbf{p}(\mathbf{q}) \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

and

$$\mathbf{R}_B^n(\mathbf{q}) \triangleq [\ \mathbf{n}(\mathbf{q}) \quad \mathbf{s}(\mathbf{q}) \quad \mathbf{a}(\mathbf{q})\ ] \ . \tag{2}$$

Let $\mathbf{D}(t)$ be the 4×4 homogeneous matrix representing the desired motion trajectory of the object with reference to the base coordinate.

To obtain the holonomic constraints for the two robots, define a "supporting orientation" as a relative orientation between the two robots or between the object and the follower in order not to drop the object. Such a supporting orientation may vary with the type of object. Two different cases are considered as follows.

Case 1 When the follower's hand has to grasp the object in order to support it (Fig. 2).

Case 2 When the follower's hand needs to support the object without grasping it (Fig. 3).

A supporting orientation for Case 1 may be obtained by the following steps (see Figs. 2 and 3).

Step 1 Suppose the equation of object surface represented in the object coordinate $(x^o,\ y^o,\ z^o)$ is given by

$$\phi(\mathbf{r}_2) = 0 \ , \quad \mathbf{r}^{\min} \le \mathbf{r}_2 \le \mathbf{r}^{\max} \ , \tag{3}$$

where $\phi : \mathbf{R}^3 \rightarrow \mathbf{R}^1$ is twice continuously differentiable and $\mathbf{r}^{\min}$ and $\mathbf{r}^{\max}$ are the 3×1 vectors representing the positional limits within which the object must lie. Then, obtain the two-dimensional surface equations of the object in the object coordinate by projecting it onto the $(x^o - z^o,\ y^o - z^o)$ plane.

Step 2 Obtain the center-line equation of the two-dimensional surfaces, and let $\mathbf{r}^* \triangleq [r_x^*\ r_y^*\ r_z^*]^T$ be the current position vector of the follower's hand with reference to the object coordinate. Then, find the derivatives $\frac{dx^o}{dz^o} \triangleq \alpha$ and $\frac{dy^o}{dz^o} \triangleq \beta$ at a position $z^0 = r_z^*$.

Step 3 Calculate the sliding vector $\mathbf{s}^*(\mathbf{r}^*)$ of the object at $\mathbf{r}^*$

$$\mathbf{s}^*(\mathbf{r}^*) = \frac{[\ \alpha \quad \beta \quad 1\ ]^T}{\sqrt{1+\alpha^2+\beta^2}} \ . \tag{4}$$

Step 4 Compute the unit vectors $\mathbf{a}^*(\mathbf{r}^*)$ and $\mathbf{n}^*(\mathbf{r}^*)$ with

$$\mathbf{a}^*(\mathbf{r}^*) \triangleq \left. \frac{\nabla\phi(\mathbf{r}_2)}{\| \nabla\phi(\mathbf{r}_2)\|} \right|_{\mathbf{r}_2 = \mathbf{r}^*} \tag{5}$$

and

$$\mathbf{n}^*(\mathbf{r}^*) \triangleq \mathbf{a}^*(\mathbf{r}^*) \times \mathbf{s}^*(\mathbf{r}^*) \ . \tag{6}$$

Then, align $\mathbf{z}_n^f$ and $\mathbf{x}_n^f$ of the follower's hand coordinate with $\mathbf{a}^*(\mathbf{r}^*)$ and $\mathbf{s}^*(\mathbf{r}^*)$, respectively.

For Case 2, a supporting orientation can be obtained by a simplified analysis of the static force equilibrium. Assume that (a) the leader's hand is capable of generating forces along the x, y, and z axes of the object coordinate by grasping the one side of object firmly, (b) the object's mass is concentrated at its mass center, and (c) the follower's hand can exert force only to the normal direction of its contact point. Also, let $\mathbf{g}$ be the gravitational acceleration vector and be represented as $[g_x, g_y, g_z]^T$ in the object coordinate, and let $\tilde{g} \triangleq [g_x, 0, 0]^T$ as shown in Fig. 4. Further, let $\mathbf{f}^l$, $\mathbf{f}^f$, M and $\mathbf{r}_1^0$ be force vectors of the leader and the follower, the mass of object, and the position vector representing the origin of (x_n^l, y_n^l, z_n^l) with respect to the origin of (x_o, y_o, z_o), respectively. Then we can obtain the following static force equilibrium eqatuion in the object coordinate:

$$\mathbf{r}_1^0 \times \mathbf{f}^l + \mathbf{r}_2 \times \mathbf{f}^f = 0 \ , \tag{7}$$

$$\mathbf{f}^l + \mathbf{f}^f + M\tilde{\mathbf{g}} = 0 \ . \tag{8}$$

Note that two other components M_{g_y} and M_{g_z} of $M\mathbf{g}$ in the object coordinate can be compensated by the corresponding counter-force component of the leader's hand force. By substituting Eq. (8) for (7), we obtain

$$\mathbf{r}_1^0 \times M\tilde{\mathbf{g}} = (-\mathbf{r}_1^0 + \mathbf{r}_2) \times \mathbf{f}^f \ . \tag{9}$$

Since we assume that the follower's hand can exert force only to the normal direction of its contact point, the force vector $\mathbf{f}^f$ of the follower can be written as

$$\mathbf{f}^f = h\mathbf{a}^*(\mathbf{r}_2), \quad h > 0 \tag{10}$$

where h is a constant representing the magnitude of the force vector. It is noted that after substituting Eq. (10) into Eq. (9), Eq. (9) becomes a nonlinear equation with an unknown vector $\mathbf{r}_2$ and an unknown constant h. If solutions to Eq. (9) exist at $\mathbf{r}_2 = \mathbf{r}^*$ and $h = h^*$, and if the follower's hand can be continuously positioned at such an $\mathbf{r}^*$, then we can compute the sliding vector $\mathbf{s}^*(\mathbf{r}^*)$. To see this, consider an example shown in Fig. 4, where x_0 is aligned with z_B, making $g_y = g_z = 0$. The object surface equation is given as $x = 0$, $-y_{\max} \le y \le y_{\max}$, and $-z_{\max} \le z \le z_{\max}$. Then, $\mathbf{a}^*(\mathbf{r}^*) = [1, 0, 0]^T$ for any $\mathbf{r}^*$. If $\mathbf{r}_1^0$ is given as $[0, r_{1y}^0, -z_{\max}]^T$, then one can obtain from Eqs. (9) and (10):

$$\mathbf{r}^* = \begin{bmatrix} r_x^* \\ r_y^* \\ r_z^* \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{-g_x r_{1y}^0 M}{h} + r_{1y}^0 \\ \frac{g_x z_{\max} M}{h} - z_{\max} \end{bmatrix} \ , \tag{11}$$

and, thus,

$$r_y^* = -\left(\frac{r_{1y}^0}{z_{\max}}\right) r_z^* \ . \tag{12}$$

In this case, $\mathbf{s}^*(\mathbf{r}^*)$ is given by

$$\mathbf{s}^*(\mathbf{r}^*) = \frac{[0, \frac{-r_{1y}^0}{z_{\max}}, 1]^T}{\sqrt{1 + (\frac{r_{1y}^0}{z_{\max}})^2}} \ . \tag{13}$$

It is remarked that Steps 1 and 2 in Case 1 can be executed off-line regardless of the movement of the object, since all the equations related to the object are defined in the object coordinate.

Now, we can obtain the following holonomic constraints; for all $t \in [t_0, t_f]$,

$$\mathbf{D}(t) = \mathbf{T}_B^n(\mathbf{q}^l)\mathbf{H}_1(\mathbf{r}_1) \tag{14}$$

and

$$\mathbf{T}_B^n(\mathbf{q}^l)\mathbf{H}_1(\mathbf{r}_1)\mathbf{H}_2(\mathbf{r}_2) = \mathbf{T}_B^n(\mathbf{q}^f) , \quad (15)$$

where t_0 and t_f are the initial and final time of the desired trajectory $\mathbf{D}(t)$, and $\mathbf{H}_1(\mathbf{r}_1)$ is a constant 4×4 matrix resulting from the prespecified distance vector $\mathbf{r}_1$, and $\mathbf{H}_2(\mathbf{r}_2)$ is given by

$$\mathbf{H}_2(\mathbf{r}_2) = \begin{bmatrix} \mathbf{s}^*(\mathbf{r}_2) & \mathbf{n}^*(\mathbf{r}_2) & \mathbf{a}^*(\mathbf{r}_2) & \mathbf{r}_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} . \quad (16)$$

In Eqs. (14) and (15), $\mathbf{q}^l$ and $\mathbf{q}^f$ are bounded as

$$\mathbf{q}^l_{\min} \le \mathbf{q}^l \le \mathbf{q}^l_{\max} \quad (17)$$

and

$$\mathbf{q}^f_{\min} \le \mathbf{q}^f \le \mathbf{q}^f_{\max} . \quad (18)$$

Now, the problem becomes: For given $\mathbf{D}(t)$, $t_o \le t \le t_f$ and $\mathbf{H}_1(\mathbf{r}_1)$, find $\mathbf{q}^l(t)$, $\mathbf{q}^f(t)$, and $\mathbf{r}_2(t)$ such that Eqs. (14-18) and (3) are satisfied for all $t \in [t_o, t_f]$.

It is here assumed that the leader has a sufficient number of dofs such that Eqs. (14) and (17) can be completely solved for all $t \in [t_o, t_f]$. Thus, we will concentrate on solving Eqs. (3), (15), (16) and (18).

In Eq. (15), the maximum number of independent constraint equations is six as discussed in [9]. Thus, if $\mathbf{r}_2$ is prespecified as in [1], and each of the two robots has six dofs, then the follower's position $\mathbf{q}^f$ can be uniquely determined by the leader's position $\mathbf{q}^l$ from the six constraint equations. However, in our case, $\mathbf{r}_2$ is a vector to be determined and, thus, the number of unknown variables could be greater than that of constraint equations. This implies that the follower be considered as a redundant manipulator with more than six dofs due to the presence of the variable $\mathbf{r}_2$.

3 SOLUTION APPROACHES

A solution to the problem formulated in Section 2 will be developed using the following steps:

- Derive the linear relationship among joint velocities and the time derivative of the supporting position vector $\mathbf{r}_2$ of the follower. This relation is necessary to solve the holonomic constraint Eqs. (3) and (15). The supporting orientation is not updated here to get a simple rotational velocity relationship between the two robots.
- Find the supporting position vector $\mathbf{r}_2$ and joint vector $\mathbf{q}^f$ of the follower by integrating $\dot{\mathbf{r}}_2 \triangleq \frac{d\mathbf{r}_2}{dt}$ and $\dot{\mathbf{q}}^f_i \triangleq \frac{d\mathbf{q}^f}{dt}$ subject to the follower's joint limits and the bounds of $\mathbf{r}_2$. The local optimization technique developed for the control of redundant manipulators [5] [6] [7] [8] is used for this.
- Recalculate the follower's joint vector $\mathbf{q}^f$ to get the desired supporting orientation with $\mathbf{r}_2$ fixed.

Note that the above solution approach does not simultaneously calculate the desired supporting orientation and $\mathbf{r}_2$ in order to avoid the computational complexity. Instead, we first find $\mathbf{r}_2$ and then calculate the supporting orientation. This is realistic if the object surface is smooth and not severely curved; otherwise, the grasping position cannot be changed while two robots are carrying an object.

3.1 Derivation of Linear Relationships Among Joint Velocities

Consider the holonomic constraint Eq. (15) which can be rewritten as

$$\mathbf{R}_B^n(\mathbf{q}^l)\mathbf{R}_1(\mathbf{r}_1)\mathbf{R}_2(\mathbf{r}_2) = \mathbf{R}_B^n(\mathbf{q}^f) \quad (19)$$

and

$$\mathbf{p}(\mathbf{q}^l) + \mathbf{R}_B^n(\mathbf{q}^l)\mathbf{r}_1 = -\mathbf{R}_B^n(\mathbf{q}^l)\mathbf{R}_1(\mathbf{r}_1)\mathbf{r}_2 + \mathbf{p}(\mathbf{q}^f) . \quad (20)$$

Eqs. (19) and (20) represent the holonomic constraints for orientation and position, respectively.

To obtain a simple rotational velocity relationship between the two robots, let

$$\mathbf{R}_2(\mathbf{r}_2) = [\, \mathbf{s}^*(\mathbf{r}^*) \quad \mathbf{n}^*(\mathbf{r}^*) \quad \mathbf{a}^*(\mathbf{r}^*) \,] \quad (21)$$

where $\mathbf{r}^*$ is the current supporting position vector. Then, from Eq. (19), we know that there is no relative motion between the two robots' end effectors. Consequently, we can establish a linear relationship between the angular velocities of the leader and the follower given by

$$\mathbf{J}_a^l(\mathbf{q}^l)\dot{\mathbf{q}}^l = \mathbf{J}_a^f(\mathbf{q}^f)\dot{\mathbf{q}}^f , \quad (22)$$

where $\mathbf{J}_a^l(\mathbf{q}^l)$ and $\mathbf{J}_a^f(\mathbf{q}^f)$ are the $3 \times n$ Jacobian matrices relating the angular velocity of the leader's and follower's end effectors with reference to base coordinate to $\dot{\mathbf{q}}^l$ and $\dot{\mathbf{q}}^f$, respectively.

To calculate the follower's velocity, differentiate Eq. (20) with respect to time. Then we can obtain

$$\left[\, \mathbf{J}_p^l(\mathbf{q}^l) + \mathbf{L}^1(\mathbf{q}^l) \,\right] \dot{\mathbf{q}}^l = \mathbf{J}_p^f(\mathbf{q}^f)\dot{\mathbf{q}}^f - \mathbf{R}_B^n(\mathbf{q}^l)\,\mathbf{R}_1(\mathbf{r}_1)\dot{\mathbf{r}}_2 - \mathbf{L}^2(\mathbf{q}^l, \mathbf{r}^*)\,\dot{\mathbf{q}}^l , \quad (23)$$

where $\mathbf{J}_p^l(\mathbf{q}^l)$ and $\mathbf{J}^f(\mathbf{q}^f)$ are another $3 \times n$ Jacobian matrices relating the positional velocities of the leader's and follower's end effectors with reference to base coordinate to $\dot{\mathbf{q}}^l$ and $\dot{\mathbf{q}}^f$, respectively, and $\mathbf{L}^1(\mathbf{q}^l)$ and $\mathbf{L}^2(\mathbf{q}^l, \mathbf{r}^*)$ are respectively given by

$$\mathbf{L}^1(\mathbf{q}^l) = \frac{\partial[\mathbf{R}_B^n(\mathbf{q}^l)\mathbf{r}_1]}{\partial \mathbf{q}_l}$$

and

$$\mathbf{L}^2(\mathbf{q}^l, \mathbf{r}^*) = \left.\frac{\partial[\mathbf{R}_B^n(\mathbf{q}^l)\mathbf{R}_1(\mathbf{r}_1)\mathbf{r}_2]}{\partial \mathbf{q}^l}\right|_{\mathbf{r}_2 = \mathbf{r}^*}$$

To impose the constraints on $\mathbf{r}_2$ as in Eq. (3) along with Eqs. (22) and (23), differentiate Eq. (3) with respect to time. Then, $\dot{\mathbf{r}}_2$ should satisfy

$$\frac{\partial \phi}{\partial t} = \left(\frac{\partial \phi}{\partial \mathbf{r}_2}\right)^T \dot{\mathbf{r}}_2 = 0 , \quad \text{for all } t \in [t_0, t_f] . \quad (24)$$

Let the moving velocity of the follower's end effector $\dot{\mathbf{r}}_2 = m\mathbf{s}^*(\mathbf{r}^*)$, where m is the magnitude of $\dot{\mathbf{r}}_2$. Recall that $\mathbf{s}^*(\mathbf{r}^*)$ has been chosen to be tangential to the object surface that the follower's hand touches. Then, Eq. (24) becomes an inactive constraint, implying that the magnitude of moving velocity m be a design variable.

By plugging Eq. (24) into Eq. (23) and combining Eq. (22) with Eq. (23), we obtain the desired linear relationship between joint velocities:

$$\mathbf{J}_f \dot{\hat{\mathbf{q}}} = \mathbf{J}_l \dot{\mathbf{q}}^l , \quad (25)$$

where the $6 \times (n+1)$ matrix $\mathbf{J}_f$, $6 \times n$ matrix $\mathbf{J}_l$, and $(n+1)$ dimensional vector $\hat{\mathbf{q}}$ are respectively given by

$$\mathbf{J}_f = \begin{bmatrix} \mathbf{J}_p^f(\mathbf{q}^f) & -\mathbf{R}_B^n(\mathbf{q}^l)\,\mathbf{R}_1(\mathbf{r}_1)\mathbf{s}^*(\mathbf{r}^*) \\ \mathbf{J}_a^f(\mathbf{q}^f) & 0 \end{bmatrix} , \quad (26)$$

$$\mathbf{J}_l = \begin{bmatrix} \mathbf{J}_p^l(\mathbf{q}^l) + \mathbf{L}^1(\mathbf{q}^l) + \mathbf{L}^2(\mathbf{q}^l, \mathbf{r}^*) \\ \mathbf{J}_a^l(\mathbf{q}^l) \end{bmatrix} , \tag{27}$$

and

$$\dot{\hat{\mathbf{q}}} = \left[(\dot{\mathbf{q}}^f)^T, m\right]^T . \tag{28}$$

Note from Eq. (25) that the number of dofs is increased by one. Thus, if the follower, like most industrial robots, has six joints, Eq. (25) plays a role of the Jacobian equation for a redundant manipulator with seven dofs. It is also noted that if $\mathbf{r}_2$ is prespecified, then $m = 0$ and, thus, Eq. (25) reduces to the Jacobian equation developed by Zheng and Luh [1].

3.2 Use of Kinematic Redundancy

In this section, the number of the follower's dofs is assumed to be six. By employing the local optimization techniques commonly used for the control of redundant manipulators, we will derive a solution for the linear Eq. (25) subject to the inequality constraints on $\mathbf{r}_2$ and $\mathbf{q}^f$ in Eqs. (3) and (18), respectively. Define $\mathbf{J}_f^\dagger$ as the generalized inverse of $\mathbf{J}_f$, i.e., $\mathbf{J}_f^\dagger \triangleq \mathbf{J}_f^T(\mathbf{J}_f\mathbf{J}_f^T)^{-1}$. Then a general solution of Eq. (25) can be given by [5]

$$\dot{\hat{\mathbf{q}}} = \mathbf{J}_f^\dagger \mathbf{J}_l \dot{\mathbf{q}}^l + (\mathbf{I} - \mathbf{J}_f^\dagger \mathbf{J}_f)\hat{\mathbf{z}} , \tag{29}$$

where $\hat{\mathbf{z}}$ is an arbitrary vector lying in the null space of $\mathbf{J}_f$. Note that Eq. (29) implies the relationship among joint velocities, but Eqs. (3) and (18) are expressed in terms of position vectors. Thus, it is necessary to change Eqs. (3) and (18) to the inequalities imposed on velocities.

To do so, let ΔT be the sampling interval and the subscript i denote the ith component of the corresponding vector. Also let

$$\rho_{\min} = \max_i \left\{ \rho_i : \rho_i = \min \left(\frac{r_i^{\min} - r_i^*}{s_i^*(\mathbf{r}^*)\Delta T} , \frac{r_i^{\max} - r_i^*}{s_i^*(\mathbf{r}^*)\Delta T} \right) \right. \tag{30}$$
$$\left. s_i^*(\mathbf{r}^*) \neq 0 , \ i = 1,2,3 \right\}$$

and

$$\rho_{\max} = \min_i \left\{ \rho_i : \rho_i = \max \left(\frac{r_i^{\min} - r_i^*}{s_i^*(\mathbf{r}^*)\Delta T} , \frac{r_i^{\max} - r_i^*}{s_i^*(\mathbf{r}^*)\Delta T} \right) \right. \tag{31}$$
$$\left. s_i^*(\mathbf{r}^*) \neq 0 , \ i = 1,2,3 \right\}$$

Here, if $s_i^*(\mathbf{r}^*) = 0$, then no more motion along the ith axis is generated. Noting that $\mathbf{r}_2$ can be approximated as

$$\mathbf{r}_2 \cong \mathbf{r}^* + m\Delta T \mathbf{s}^*(\mathbf{r}^*) , \tag{32}$$

Eq. (3) is modified to

$$\rho_{\min} \leq m \leq \rho_{\max} . \tag{33}$$

Similarly, using the first order approximation of $\dot{\mathbf{q}}^f$, the inequality constraints on $\mathbf{q}^f$ can be modified to

$$\delta\mathbf{q}_{\min} \leq \dot{\mathbf{q}}^f \leq \delta\mathbf{q}_{\max} , \tag{34}$$

where $\delta\mathbf{q}_{\min}$ and $\delta\mathbf{q}_{\max}$ are given by

$$\delta\mathbf{q}_{\min} \triangleq \frac{\mathbf{q}_{\min}^f - \mathbf{q}^f}{\Delta T} , \tag{35}$$

and

$$\delta\mathbf{q}_{\max} \triangleq \frac{\mathbf{q}_{\max}^f - \mathbf{q}^f}{\Delta T} . \tag{36}$$

To find $\hat{\mathbf{z}}$ such that the inequality constraints (33) and (34) are satisfied, let the cost function $\mathbf{I}(\dot{\hat{\mathbf{q}}})$ be chosen as

$$I(\dot{\hat{\mathbf{q}}}) = \| \dot{\hat{\mathbf{q}}} - \delta\hat{\mathbf{q}}_c \|_{\mathbf{S}}^2 . \tag{37}$$

$\mathbf{S}$ is a 7×7 positive definite weighting matrix and $\delta\hat{\mathbf{q}}_c$ is a 7×1 vector defined by

$$\delta\hat{\mathbf{q}}_c \triangleq [\ \delta q_{1c}\ \delta q_{2c} \cdots \delta q_{6c}\ \rho_c\ ] , \tag{38}$$

where δq_{ic}, $i = 1, 2, \ldots, 6$ and ρ_c are given by

$$\delta q_{ic} = \frac{\delta q_{i,\max} + \delta q_{i,\min}}{2} \qquad i = 1, 2, \cdots, 6 \tag{39}$$

and

$$\rho_c = \frac{\rho_{\max} + \rho_{\min}}{2} . \tag{40}$$

Such a $\hat{\mathbf{z}}$ can be found by minimizing $I(\dot{\hat{\mathbf{q}}})$ in Eq. (37) subject to Eq. (29), since minimization of $I(\dot{\hat{\mathbf{q}}})$ implies that all the joint variables and $\mathbf{r}_2$ be located at the centers of their motion ranges. The solution vector $\hat{\mathbf{z}}$ to the minimization of $I(\dot{\hat{\mathbf{q}}})$ subject to Eq. (29) is given by

$$\hat{\mathbf{z}} = \left[(\mathbf{I} - \mathbf{J}_f^\dagger \mathbf{J}_f)\, \mathbf{S} (\mathbf{I} - \mathbf{J}_f^\dagger \mathbf{J}_f) \right]^\dagger (\mathbf{I} - \mathbf{J}_f^\dagger \mathbf{J}_f)\, \mathbf{S} (\delta\hat{\mathbf{q}}_c - \mathbf{J}_f^\dagger \mathbf{J}_l \dot{\mathbf{q}}^l) . \tag{41}$$

3.3 Resolved Motion for Desired Supporting Orientation

It may be necessary to resolve the follower's joint vector $\mathbf{q}^f$ into the desired supporting orientation when the supporting position vector $\mathbf{r}_2$ is updated. This can be done easily by solving the follower's Jacobian equation as follows: first, obtain the required differential change $\delta w_x, \delta w_y$ and δw_z of the hand orientation by employing two rotation matrices $\mathbf{R}_B^n(\mathbf{q}^l)\mathbf{R}_1(\mathbf{r}_1)\mathbf{R}_2(\mathbf{r}_2^*)$ and $\mathbf{R}_B^n(\mathbf{q}^l)\mathbf{R}_1(\mathbf{r}_1)\mathbf{R}_2(\mathbf{r}_2)$ as in [9]. Then, find the corresponding differential change of the joint vector $\delta\mathbf{q}^f$ by

$$\delta\mathbf{q}^f = \begin{bmatrix} \mathbf{J}_p(\mathbf{q}_f) \\ \mathbf{J}_a(\mathbf{q}_f) \end{bmatrix}^{-1} [0\ 0\ 0\ \delta w_x\ \delta w_y\ \delta w_z]^T .$$

Finally, obtain the desired $\mathbf{q}^f$ by adding $\delta\mathbf{q}^f$ to the old $\mathbf{q}^f$.

It is remarked that the updated $\mathbf{q}^f$ may not satisfy the constraints (18). In such a case, the weighting matrix $\mathbf{S}$ in Eq. (37) may have to be adjusted such that heavier weights are placed on those joints violating the limits in this updating procedure.

4 A Numerical Example

Two PUMA 560 manipulators, each of which has six rotational joints are employed to demonstrate the utility and power of the proposed coordination method. The task given to the two arms is to transfer an 850 mm long rigid bar shown in Fig. 4 along a straight line path. Numerical values used in our simulation are:

1. Link parameters and joint ranges of the PUMA 560 manipulator are listed in Table 1.

2. Initial joint values and initial grasping position of the follower are tabulated in Table 2.

3. $\mathbf{r}_{min}$ and $\mathbf{r}_{max}$ are chosen to be $[0,0,0]^T$ and $[0 \quad 0 \quad 400]^T$, respectively.

4. The sampling interval ΔT is chosen to be 31 msec.

5. The desired trajectory $\mathbf{D}(t)$ with reference to the $0th$ coordinate frame of the leader is given by

$$\mathbf{D}(t) = \begin{bmatrix} 0 & 1 & 0 & d_x(t) \\ 0 & 0 & 1 & 690 \\ 1 & 0 & 0 & 260.4 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

where $d_x(t) = \frac{t}{31} - 0.3$.

6. The 7×7 weighting matrix $\mathbf{S}$ is chosen to be a diagonal matrix given by

$$\mathbf{S} = diag(1,1,1,1,1,1,s_7) ,$$

where s_7 is a constant representing the weighting for the grasping position.

Note that these numerical values are chosen arbitrarily and the capability of our method is independent of the choice of these values.

It is remarked that the sliding vector $\mathbf{s}^*(\mathbf{r}^*)$ in Eq. (4) is easily obtained as $[0 \quad 0 \quad 1]^T$ and fixed during the entire motion for the task. Thus, the supporting orientation in this example does not change with time during the entire task.

Fig. 5 shows position trajectories of joint 3 of the follower for the following three cases.

Case (a): When the grasping position vector $\mathbf{r}_2$ of the follower is fixed as in [1] and chosen as $[0 \quad 0 \quad 450]^T$,

Case (b): When the proposed method is applied with s_7 equal to 100,

Case (c): When the proposed method is applied with s_7 equal to 1000.

From Fig. 5, it is observed that in case (a), a portion of joint trajectory violated the joint limit, while in cases (b) and (c), joint trajectories lie within the specified joint ranges as a result of moving the follower's hand toward the 3/4 position of the bar. Changes in the grasping position of cases (b) and (c) are shown in Fig. 6. It is noted from Figs. 5 and 6 that the initial joint motion in case (c) is smaller than that in case (b). Thus, we may expect that if the initial movement of the follower's hand is large enough to cause the violation of the limits of joint velocity $\dot{\mathbf{q}}^f_{min}$ and $\dot{\mathbf{q}}^f_{max}$, then such violations can be avoided by adjusting the weighting matrix $\mathbf{S}$ together with $\mathbf{r}_{min}$ and $\mathbf{r}_{max}$ and/or by modifying Eq. (34) as

$$\max(\delta\mathbf{q}_{min}, \dot{\mathbf{q}}^f_{min}) \leq \dot{\mathbf{q}}^f \leq \min(\delta\mathbf{q}_{max}, \dot{\mathbf{q}}_{max}) .$$

This example has shown that our coordination method is more powerful and flexible than those methods in [1, 2, 3, 4].

5 Concluding Remarks

A new method for coordinating dual robot arms was proposed, where allowing the follower to change its grasping position was equivalent to adding one dof to the follower without actual installation of an additional joint motor. The method was then mathematically and numerically shown to be effective for the task of carrying a solid object along a prespecified path while avoiding the joint limits. It is worth mentioning that our method can be considered as a generalization of Zheng and Luh's work [1] and can be further generalized by allowing not only the follower but also the leader to change their grasping position and orientation.

As a future work, dynamics of dual arms holding and/or supporting a common object need to be investigated to deal with the constraints of joint torques.

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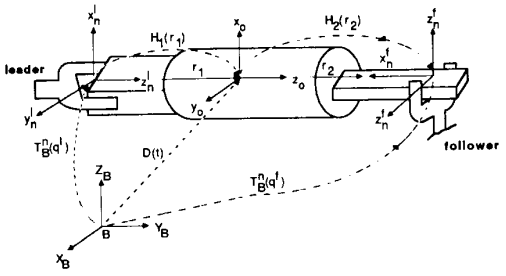

Figure 1: Two robots handling a rigid body object.

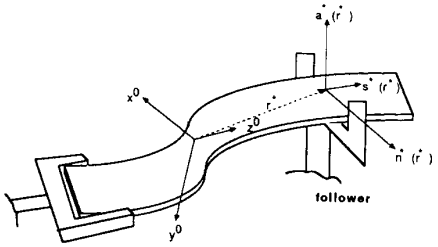

Figure 2: Supporting orientation when a robot grasps an object.

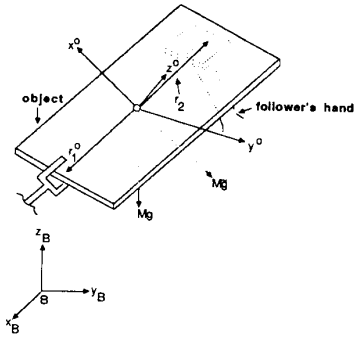

Figure 3: Supporting orientation when the follower does not have to grasp the object.

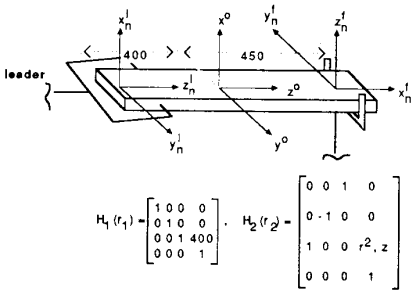

Figure 4: A 850 mm long rigid bar and hand orientations of the leader and the follower.

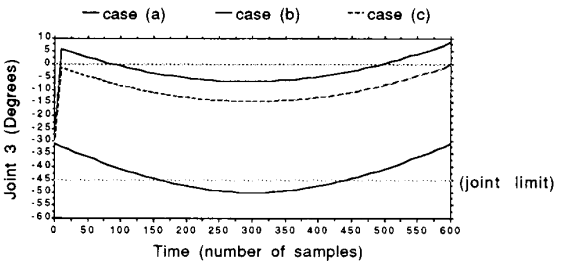

Figure 5: Position trajectories of Joint 3 of the follower.

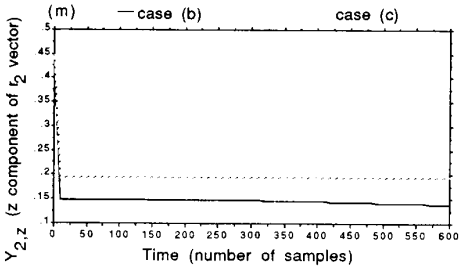

Figure 6: Change in the gripping position.

Joint i	α_i (degree)	a_i (mm)	d_i (mm)	Joint range (degree)
1	-90	0	0	-160 to +160
2	0	431.8	149.09	-225 to 45
3	90	-20.32	0	-45 to 225
4	-90	0	433.07	-110 to 170
5	90	0	0	-100 to 100
6	0	0	56.25	-266 to 266

Table 1. Link parameters and joint range of PUMA robot.

q_1	q_2	q_3	q_4	q_5	q_6	$r_{2,z}$
-164.1774	32.065	-30.9446	0	-1.1204	-105.8256	450

Table 2. Initial joint values and gripping position of the follower.