On Non-existence of Optimal Local Synchronous Bandwidth Allocation Schemes for the Timed-Token MAC Protocol

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Abstract

Numerous methods have been proposed to integrate real-time and non-real-time services of the timed-token medium access control (MAC) protocol. One of the key issues in tailoring the timed-token protocol for real-time applications is the synchronous bandwidth allocation (SBA) problem whose objective is to meet both the protocol and deadline constraints. Several non-optimal local SBA schemes and two optimal global schemes have been proposed [1-4]. Local SBA schemes use only information available locally to each node, and are thus preferred to global schemes because of their lower network-management overhead. However, we formally prove, using the technique of adversary argument, that there does not exist any optimal local SBA scheme. During the preparation for this proof, we also derive a timing property that generalizes the previous findings about the cycle-time properties of the timed-token protocol.

1 Introduction

There has been an increasing need of timely and dependable communication services for such real-time systems as multimedia, automated factories, and industrial process controls. To meet this need, network architectures and protocols are required to provide users with a convenient means of guaranteeing message-transmission delay bounds. Solutions to this problem will not only improve the quality of service, but also expand their application domains to distributed real-time controls and digital continuous-media (motion video or audio) transmissions.

The problem of guaranteeing the timely delivery of messages has been studied by numerous researchers. Their efforts have been directed mainly towards designing medium access control protocols for multi-access networks which deliver messages with timing constraints. Among all the methods designed to integrate real-time and non-real-time applications, the timed-token MAC protocol has attracted considerable attention because of its bounded access time. The timed-token protocol groups messages into two classes: synchronous and asynchronous. Synchronous messages arrive at regular intervals and are usually associated with delivery deadlines. Asynchronous messages have no such time constraints. At network initialization, a protocol parameter called the *Target Token Rotation Time* (TTRT) is negotiated among the nodes/stations to specify the expected token rotation time. Each node i is assigned a portion H_i of TTRT as its *synchronous bandwidth*, which is the maximum time node i is permitted to transmit its synchronous messages every time it receives the token. The assignment of H_i is also subject to the *protocol constraint* that the total bandwidth allocated for synchronous traffic over all nodes should not exceed TTRT (minus various protocol-dependent overheads). Whenever a node receives the token, it transmits its synchronous messages, if any, up to H_i units of time. The node can transmit its asynchronous messages only if the time interval between the previous token arrival and the current token arrival is less than TTRT, i.e., the token arrived *earlier* than expected.

Many researchers studied the access time bounds and other timing properties of the timed-token protocol. In particular, Johnson *et al.* [5,6] prove that the *average* token cycle time is bounded by TTRT, and the *maximum* token cycle time is bounded by 2 $\times$ TTRT. Agrawal *et al.* [1,2] extend Johnson's result and prove that the time elapsed between k consecutive token's visits to a node is bounded by k $\times$ TTRT. They also formulated a *synchronous bandwidth allocation* (SBA) problem and attempted to calculate the synchronous bandwidth H_i that should be allocated to node i, for all i, to meet the protocol constraint and transmit all synchronous messages before their deadlines. Succinctly, H_i should be assigned so that the minimum time available for node i to transmit a synchronous message after its arrival but before its delivery deadline is greater than or equal to the worst-case message transmission time. This timing constraint in calculating H_i's is called the *deadline constraint*.

As discussed in [1], SBA schemes can be classified as local or global. A local SBA scheme uses only information available locally to node i, i.e., the parameters of a synchronous message stream at node i. On the other hand, in a global scheme, each node i uses the parameters of all nodes' synchronous message streams to compute H_i. The extra information on other nodes used by a global scheme may help it find better values of H_i's. However, any change in a node's message stream parameters may require the global scheme to adjust the synchronous bandwidths of *all* nodes. By contrast, in a local scheme, if the message stream parameters of node i change, only H_i needs to be re-calculated and H_j's, $j \neq i$, need not be modified because they were calculated

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independently of node i's message stream parameters.

As the global SBA schemes use global information to allocate synchronous bandwidths, they are naturally expected to achieve better performance. To our best knowledge, there are only two optimal global SBA schemes [2,3] and several non-optimal local SBA schemes [1,4] reported in the open literature. By an "optimal" SBA scheme, we mean a SBA scheme that finds a feasible set of H_i's subject to the protocol and deadline constraints whenever such a set exists. Whether to choose a non-optimal local scheme or an optimal global scheme depends on the trade-off between the ease of network management and the resulting performance improvement. One important remaining issue is to determine if there exists any optimal local SBA scheme. In this paper, we formally prove, using the adversary argument technique, that there doesn't exist any optimal local SBA scheme. In the course of preparing for this proof, we also derive a timing property that generalizes both Johnson's and Agrawal's results.

The rest of the paper is organized as follows. In Section 2, we discuss the synchronous message model used for real-time applications and give a brief overview of the timed-token MAC protocol. In Section 3, we present several timing properties for the timed-token MAC protocol and discuss the timing requirements imposed on the protocol by the messages with delivery deadlines. In Section 4, we formulate the SBA problem and then present the proof of non-existence of optimal local SBA schemes. We conclude the paper with Section 5.

2 Message model and MAC protocol

In this section, we first discuss the synchronous message model suitable for real-time applications. To make the paper self-contained, we also review the timed-token MAC protocol used in FDDI token rings and some of its timing properties. A more detailed description of the timed-token protocol and FDDI token rings can be found in [7,8].

2.1 Message model

Let n be the number of nodes/stations in the system. Without loss of generality, we assume that there is one synchronous message stream at each node. (As was discussed in [1], a more general token ring network in which a node has more than one synchronous message stream can be transformed into an equivalent network with one synchronous message stream at each node.) The message stream at node i can be described by a triple (C_i, D_i, P_i), where

- P_i is the minimum inter-arrival period for the message stream at node i, i.e., if the j-th message arrives at node i at time t, then the $(j+1)$-th message will arrive at time $t + P_i$ or later, where $j \geq 1$,
- C_i is the maximum message transmission time at node i, i.e., C_i is the time needed to transmit a maximum-size message, and
- D_i is the *relative* deadline for the message stream at node i, i.e., if a message arrives at node i at time t, then it must be transmitted by time $t + D_i$.

The objective of an SBA scheme is to properly set the parameters of the MAC protocol so as to guarantee the delivery of each message in node i's synchronous message stream within a time period $\leq D_i$ after its arrival, as long as the message inter-arrival time is $\geq P_i$ and the message transmission time is $\leq C_i$.

2.2 MAC protocol

The key idea of the timed-token MAC protocol is to control the token rotation time. A protocol parameter called the *target token rotation time* (TTRT) is determined upon network initialization, and specifies the expected token rotation time. The TTRT is chosen to be sufficiently small so that the responsiveness requirements at every node may be met.

Each node i is assigned a portion H_i of TTRT, known as its *synchronous bandwidth*, which is the maximum time a node is permitted to transmit synchronous messages every time it receives the token. The token is then forced by the protocol to circulate with sufficient speed so that all nodes receive their allocated fractions of bandwidth for transmitting synchronous messages. This is achieved by transmitting asynchronous messages only when the token rotates sufficiently fast so that it returns to a node within the TTRT, i.e., it arrives *early*. Specifically, each node has two timers and one counter:

- The *token rotation timer* (TRT) records the time elapsed since the last token's visit (if the TRT has not yet expired).
- The *token holding timer* (THT) records the amount of time by which the token has arrived early, i.e., the amount of time which can be used to transmit asynchronous messages.
- The *late counter* (LC) records the number of times its TRT has expired since the last token's visit.

After the TTRT value is negotiated among the nodes during the network initialization, each node i initializes its timers and counter as follows:

$$\text{TRT} \leftarrow \text{TTRT}; \ \text{THT} \leftarrow 0; \ \text{LC} \leftarrow 0.$$

TRT is enabled during all ring operations and counts down until one of the following three events occurs:

E1. TRT reaches zero: The following steps are taken: (i) TRT $\leftarrow$ TTRT, and TRT continues to count down, and (ii) LC $\leftarrow$ LC + 1.

E2. The token arrives early: That is, when the token arrives, the time elapsed since the previous token's visit is less than TTRT (LC = 0). In this case, the following steps are taken: (i) THT $\leftarrow$ TRT, and THT counts down only during the transmission of asynchronous messages, (ii) TRT $\leftarrow$ TTRT, and TRT continues to count down, (iii) asynchronous messages, if any, are transmitted until THT expires or until all asynchronous messages are transmitted, whichever occurs first, and (iv) synchronous messages are transmitted up to H_i units of time or until all synchronous messages are transmitted, whichever occurs first.[1]

E3. The token arrives late: That is, LC $\neq 0$ at the time of token arrival. In this case, the following steps are taken: (i) LC $\leftarrow 0$, (ii) TRT is not reset, and continues to count down, and (iii) only synchronous messages can be transmitted up to H_i units of time, and no asynchronous messages can be transmitted.

3 Protocol timing properties and real-time requirements

In this section, we discuss several interesting timing properties associated with the MAC protocol and the timing requirements imposed by the messages with delivery deadlines on the MAC protocol.

To facilitate the discussion and the subsequent derivation, we will use the *circular sum* operator defined in [6]:

$$\sum_{i,j=c,k}^{\ell,m} p_{i,j} = \sum_{j=k}^{n} p_{c,j} + \sum_{i=c+1}^{\ell-1} \sum_{j=1}^{n} p_{i,j} + \sum_{j=1}^{m} p_{\ell,j},$$

where $1 \leq k, m \leq n$, and n is the number of nodes in the network, and

$$\sum_{i=j}^{k} p_i = \begin{cases} p_j + p_{j+1} + \cdots + p_k, & \text{if } j \leq k \leq n; \\ p_j + \cdots + p_n + p_1 + \cdots + p_k, & \text{if } k < j \leq n. \end{cases}$$

Note that the definition of the circular sum operator results from the fact that all the nodes are in either a logical or a physical ring. We define the following notation:

- T: the TTRT of an FDDI network.
- θ_i: the latency between node i and its upstream neighbor. θ_i includes medium propagation delay, token transmission time, station latency, and token capture delay [6].
- Θ: the *ring latency*, i.e., $\Theta = \sum_{i=1}^{n} \theta_i$.
- Ω: the various protocol-dependent overheads.
- τ: the portion of the synchronous bandwidth unavailable for transmitting synchronous messages, i.e., $\tau = \Theta + \Omega$.
- $\vec{H}$: vector $(H_1, H_2, \ldots, H_n)$, where H_i is the synchronous bandwidth allocated to node i.
- $g_{c,i}$ ($a_{c,i}$): the time spent on transmitting synchronous (asynchronous) traffic on the c-th token's visit to node i.
- $C_{c,i}$: the length of the complete token rotation that ends with the c-th visit to node i. By using the circular sum operator, $C_{c,i}$ can be expressed as

$$C_{c,i} = \sum_{j,k=c-1,i+1}^{c,i} (g_{j,k} + a_{j,k}) + \tau.$$

[1]In the MAC protocol, it is not specified which of synchronous or asynchronous traffic will be transmitted first.

- $d_i(\ell)$: the time when the token departs from node i the ℓ-th time, i.e., the time when node i finishes the transmission of its synchronous and/or asynchronous messages, if any, and starts the transmission of the token to its downstream neighbor the ℓ-th time.
- X_i: the minimum time available for node i to transmit synchronous messages in an interval $(t, t + D_i]$.
- f_g, f_l: the functions which represent the global and local synchronous bandwidth allocation schemes, respectively. That is, a global allocation scheme can be represented as $\vec{H} = f_g(\vec{C}, \vec{D}, \vec{P}, T, \tau)$, where $\vec{C} = (C_1, C_2, \ldots, C_n)$, $\vec{D} = (D_1, D_2, \ldots, D_n)$, and $\vec{P} = (P_1, P_2, \ldots, P_n)$. A local allocation scheme can be represented as $H_i = f_l(C_i, D_i, P_i, T, \tau)$, for $i = 1, 2, \ldots, n$.

Note that a node i can transmit its synchronous messages only up to its assigned synchronous bandwidth H_i, and can transmit its asynchronous messages only when the token arrives early and only up to the amount of time by which the token arrived early. So, for $c \geq 1$ and $1 \leq i \leq n$,

$$\begin{aligned} g_{c,i} &\leq H_i; \\ a_{c,i} &\leq \max(0, T - C_{c,i-1}). \end{aligned}$$

Timing properties of the protocol: The *protocol constraint* on the allocation of synchronous bandwidth states that the total bandwidth allocated to synchronous traffic among all nodes in a timed-token ring should not exceed the available portion $T - \tau$ of TTRT:

$$\sum_{i=1}^{n} H_i \leq T - \tau. \tag{3.1}$$

Violation of the protocol constraint will make the ring unstable and oscillate between "claiming" and "operational."

Let $\Delta_{b,i}(\ell, c)$ be the time difference between a reference time point $d_b(\ell)$ and the time when the token departs from node i the c-th time *after* $d_b(\ell)$. That is,

$$\Delta_{b,i}(\ell, c) = \begin{cases} d_i(\ell + c - 1) - d_b(\ell) & \text{if } 1 \leq b < i \leq n; \\ d_i(\ell + c) - d_b(\ell) & \text{if } 1 \leq i \leq b \leq n. \end{cases}$$

Under the protocol constraint, the following well-known result is formally proved in [5,6].

Theorem 1: (Johnson and Sevcik's Theorem) For the timed-token MAC protocol, the worst-case token rotation time — the time interval between the ℓ-th token's departure and the $(\ell + 1)$-th token's departure from node b — is bounded by $T + (\sum_{j=1}^{n} H_j + \tau)$, i.e.,

$$\Delta_{b,b}(\ell, 1) = d_b(\ell + 1) - d_b(\ell) \leq T + \sum_{j=1}^{n} H_j + \tau \leq 2 \cdot T,$$

for any $1 \leq b \leq n$, and $\ell \geq 1$. □

An important lemma follows from Theorem 1:

Lemma 1: For the timed-token MAC protocol,

$$\Delta_{b,i}(\ell,1) \le T + \sum_{j=b+1}^{i} (H_j+\theta_j) + \Omega \le T + \sum_{j=b+1}^{i} H_j + \tau,$$

for any $1 \le b \le n$ and $\ell \ge 1$.

Proof: We prove the lemma for the case of $1 \le b < i \le n$. The proof for the case of $1 \le i \le b \le n$ is similar, and thus omitted. The proof is by contradiction. Assume that $\Delta_{b,i}(\ell,1) > T + \sum_{j=b+1}^{i}(H_j+\theta_j) + \Omega$. Then,

$$\begin{aligned}\Delta_{b,b}(\ell,1) &= d_b(\ell+1) - d_b(\ell) \\ &= (d_b(\ell+1) - d_i(\ell)) + (d_i(\ell) - d_b(\ell)) \\ &> (d_b(\ell+1) - d_i(\ell)) + T \\ &\quad + \sum_{j=b+1}^{i} (H_j+\theta_j) + \Omega. \end{aligned} \quad (3.2)$$

If the synchronous bandwidths are fully utilized during the ℓ-th token's visits to node $i+1,\ldots,n$ and during the $(\ell+1)$-th token's visits to node $1,\ldots,b$, i.e., $d_b(\ell+1) - d_i(\ell) \ge \sum_{j=i+1}^{b}(H_j+\theta_j)$, then $d_b(\ell+1) - d_b(\ell) > T + \sum_{j=1}^{n}(H_j + \theta_j) + \Omega = T + \sum_{j=1}^{n} H_j + \tau$ in Eq. (3.2), contradicting Theorem 1. This contradiction implies that our assumption $d_i(\ell) - d_b(\ell) > T + \sum_{j=b+1}^{i}(H_j+\theta_j) + \Omega$ must be false. □

Following similar derivation steps as in [6], we can obtain a more general result on the upper bound of the time difference between $d_b(\ell)$ and the time when the token leaves node i the c-th time after $d_b(\ell)$:

Theorem 2: For the timed-token MAC protocol,

$$\begin{aligned}\Delta_{b,i}(\ell,c) &\le c\cdot T + \sum_{j=b+1}^{i} (H_j+\theta_j) + \Omega \\ &\le c\cdot T + \sum_{j=b+1}^{i} H_j + \tau \le (c+1)\cdot T, \end{aligned} \quad (3.3)$$

for any $\ell \ge 1$ and $c \ge 1$.

Proof: We prove the theorem for the case of $1 \le i < b \le n$. The proof for the other case is similar, and thus omitted. Let $Q_{c,i} = c\cdot T + \sum_{j=b+1}^{i}(H_j+\theta_j) + \Omega$ and $R_{c,i} = \Delta_{b,i}(\ell,c) = d_i(\ell+c) - d_b(\ell)$ (for the case of $1 \le i < b \le n$), and let $G_{c,i} \triangleq Q_{c,i} - R_{c,i}$. We want to show that $G_{c,i} \ge 0$.

The proof is by contradiction. Assume that the x-th token's visit to node y after time $d_b(\ell)$ is the first visit after $d_b(\ell)$ for which $G_{c,i}$ is negative. Then $G_{x,y} < 0$, but $G_{j,k} \ge 0$ for $1, b+1 \le j,k < x,y$. First, x must be ≥ 2, because

$$G_{1,i} = (T + \sum_{j=b+1}^{i} (H_j+\theta_j) + \Omega) - \Delta_{b,i}(\ell,1) \ge 0,$$

where the inequality comes from Lemma 1. Now, we consider two cases:

Case 1: $g_{\ell+x,y} + a_{\ell+x,y} \le H_y$. Consider the relationship between $G_{x,y}$ and $G_{x,y-1}$:

$$\begin{aligned} & G_{x,y} - G_{x,y-1} \\ =\; & (Q_{x,y} - Q_{x,y-1}) - (R_{x,y} - R_{x,y-1}) \\ =\; & (H_y + \theta_y) - (d_y(\ell+x) - d_{y-1}(\ell+x)) \\ =\; & H_y - (g_{\ell+x,y} + a_{\ell+x,y}) \ge 0. \end{aligned}$$

Hence $G_{x,y} \ge G_{x,y-1}$.

Case 2: $g_{\ell+x,y} + a_{\ell+x,y} > H_y$. Since $H_y \ge g_{\ell+x,y}$, we know that $a_{\ell+x,y} > 0$ in this case. That is, the $(\ell+x)$-th token's visit to node y occurs early, or $C_{\ell+x,y-1} = \sum_{j,k=\ell+x-1,y}^{\ell+x,y-1}(g_{j,k} + a_{j,k}) + \tau < T$, and hence

$$0 < a_{\ell+x,y} \le \max(0, T - C_{\ell+x,y-1}) = T - C_{\ell+x,y-1}.$$

Consider the relationship between $G_{x,y}$ and $G_{x-1,y-1}$:

$$\begin{aligned} & G_{x,y} - G_{x-1,y-1} \\ =\; & (Q_{x,y} - Q_{x-1,y-1}) - (R_{x,y} - R_{x-1,y-1}) \\ =\; & (T + H_y + \theta_y) - (d_y(\ell+x) - d_{y-1}(\ell+x-1)) \\ =\; & T + H_y - (C_{\ell+x,y-1} + g_{\ell+x,y} + a_{\ell+x,y}) \\ =\; & [(T - C_{\ell+x,y-1}) - a_{\ell+x,y}] + (H_y - g_{\ell+x,y}) \ge 0. \end{aligned}$$

Hence $G_{x,y} \ge G_{x-1,y-1}$.

We showed that $G_{x,y}$ was no less than $G_{x,y-1}$ in **Case 1** and no less than $G_{x-1,y-1}$ in **Case 2**, implying that x,y was not the first visit for which $G_{x,y}$ is negative. This contradiction shows that our assumption must be false, and thus the theorem is proved. □

Note that if $\alpha_i(\ell)$ denotes the time of the token's ℓ-th arrival at node i, then for $i < n$, $d_i(\ell) = \alpha_{i+1}(\ell)$, and for $i = n$, $d_n(\ell) = \alpha_1(\ell+1)$.[2] Therefore, it is easy to see that results similar to the above lemma/theorems can be derived for token arrival times.

If we set $b = i$ in Eq. (3.3), we obtain the following corollary.

Corollary 1: For the timed-token protocol, the time elapsed between any $c+1$ consecutive token's visits to a node is bounded by $c\cdot T + \sum_{j=1}^{n} H_j + \tau \le (c+1)\cdot T$. □

A similar result of Corollary 1 was also obtained by Agrawal *et al.* [1,9] using a more complicated approach. An example showing that the bound is tight can also be found in [9].

The deadline constraint: Every synchronous message for real-time applications must be transmitted before its delivery deadline. That is, the minimum amount of time, X_i, available for node i to transmit its synchronous messages in an interval $(t, t+D_i]$ should be no less than the required maximum message transmission time. Using Corollary 1, Agrawal *et al.* [1,2] derived a lower bound for the time available for a node to transmit its synchronous messages within a given time period D_i as follows.

Theorem 3: Assume that at time t, a synchronous message with deadline D_i arrives at node i $(1 \le i \le n)$. Then, the minimum amount of time, X_i, available for node i to transmit this synchronous message before its deadline is given by

$$X_i(\vec{H}) = (q_i - 1)\cdot H_i + \max(0, \min(r_i - (\sum_{\substack{j=1,\ldots,n \\ j\ne i}} H_j + \tau), H_i)). \quad (3.4)$$

where $q_i = \lfloor D_i/T \rfloor$ and $r_i = D_i - q_i \cdot T$. □

[2] Assume that latency/overhead is ignored.

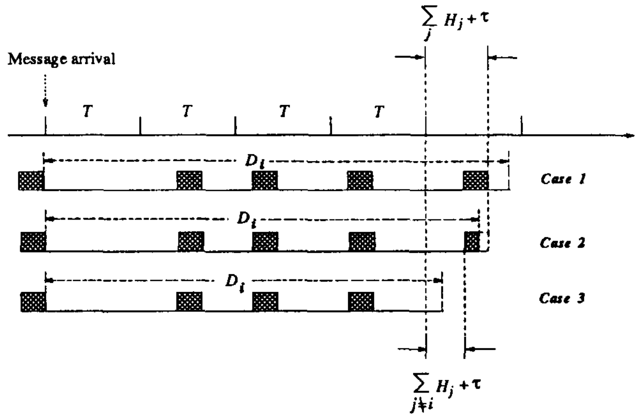

Figure 1: Worst-case token visit scenarios.

Note that Eq. (3.4) can be re-written as

$$X_i(\vec{H}) = \begin{cases} (1)\ q_i \cdot H_i, \text{ if } r_i \geq \sum_j H_j + \tau, \\ (2)\ (q_i - 1) \cdot H_i + r_i - \sum_{j \neq i} H_j - \tau, \\ \quad \text{if } \sum_{j \neq i} H_j + \tau < r_i < \sum_j H_j + \tau, \\ (3)\ (q_i - 1) \cdot H_i, \text{ if } r_i \leq \sum_{j \neq i} H_j + \tau. \end{cases} \tag{3.5}$$

Also, note that the time available for a node to transmit a synchronous message before its deadline becomes *minimal* when the message arrives *right after* the token's departure. Fig. 1 depicts the scenarios where Case 1, 2, or 3 may arise. For $D_i \leq P_i$, the timing requirements of synchronous messages impose the deadline constraint that $X_i(\vec{H}) \geq C_i$, for $i = 1, 2, \ldots, n$.

4 SBA problem and non-existence proof

In this section, we give a proof on the non-existence of optimal local SBA schemes of the form $H_i = f_l(C_i, D_i, P_i, T, \tau)$, for $i = 1, 2, \ldots, n$. Without loss of generality, it suffices for us to prove this claim for the special case of $D_i = P_i$, for all i, and $\tau = 0$. P_i and τ are thus dropped in the following discussion.

Problem 1: (The SBA Problem) Given the number of nodes (or synchronous message streams), n, the maximum message transmission time vector, $\vec{C} = (C_1, C_2, \ldots, C_n)$, the deadline vector, $\vec{D} = (D_1, D_2, \ldots, D_n)$, and the negotiated TTRT, T, the objective of a global SBA scheme is to find an algorithm that realizes the function f_g:

$$\vec{H} = (H_1, H_2, \ldots, H_n) = f_g(\vec{C}, \vec{D}, T), \tag{4.1}$$

and the objective of a local SBA scheme is to find an algorithm that realizes the function f_l:

$$H_i = f_l(C_i, D_i, T), \text{ for } i = 1, 2, \ldots, n, \tag{4.2}$$

subject to

$$\textbf{protocol constraint:} \quad \sum_{i=1}^{n} H_i \leq T - \tau, \tag{4.3}$$

$$\textbf{deadline constraint:} \quad X_i(\vec{H}) \geq C_i, \tag{4.4}$$

where $X_i(\vec{H})$ is defined in Eq. (3.4). □

A *feasible* solution for the above SBA problem is a vector $\vec{H}$ that satisfies both the protocol and deadline constraints. An *optimal* global (local) SBA scheme is the one that realizes the function f_g (f_l) whenever such a solution exists.

As mentioned in Section 1, whether or not there exists any optimal local SBA scheme remains unknown. The following theorem provides a definite answer to this issue.

Theorem 4: There does *not* exist any optimal local scheme for the SBA problem.

Proof: For clarity of presentation, we outline the skeleton of the proof here and leave the detailed algebraic manipulation in the Appendix. Our proof is based on the technique of *adversary argument*, a detailed account of which can be found in [10].

Let L be any local SBA scheme, and let A be the adversary. A first chooses (and fixes) the values for C_1, D_1, and T, and asks L for the value of H_1. Since L is a local SBA scheme, it should be able to give a value of H_1, say h, based only on the values of C_1, D_1, and T. After L gives A the value h of H_1, A chooses the values for n, C_i, and D_i, for $i = 2, 3, \ldots, n$, such that with $H_1 = h$ given by L, it is impossible to find a feasible solution $\vec{H} = (h, H_2, H_3, \ldots, H_n)$ for the SBA problem with the chosen n, T, C_i, D_i, for $i = 1, 2, \ldots, n$. However, a feasible solution does exist if H_1 is not restricted to be h. If for every value h that A receives from L, A can always design an instance of the SBA problem such that the above situation occurs, then by the adversary argument, we prove that L cannot be an optimal local SBA scheme since there are cases in which feasible solutions exist but L is not able to find one. Theorem 4 is thus proved.

We now discuss how A chooses the instances for the SBA problem and jeopardizes the optimality claim of any local SBA scheme L.

A first chooses:

$$C_1 = \frac{1}{3}T + \delta \text{ and } D_1 = 3T - (\frac{1}{3}T + \delta),$$

where T is the TTRT and can be any fixed positive number, and δ is a positive number in $(0, \frac{1}{15}T]$. (The reason for choosing δ in $(0, \frac{1}{15}T]$ is given in Appendix A). Suppose L computes a value h of H_1 based on the given values of C_1, D_1, and T. We consider two cases for the h value L computes, one for $h > \frac{1}{3}T$ and the other for $h \leq \frac{1}{3}T$.

C1: $h > \frac{1}{3}T$: A can choose

$$n = 3, C_2 = C_3 = C_1, \text{ and } D_2 = D_3 = D_1.$$

Since all C_i's are equal, all D_i's are equal, and L is a local scheme, L will compute $H_2 = H_3 = H_1 = h > \frac{1}{3}T$. Since $\sum_{i=1}^{3} H_i = 3h > T$ and the protocol constraint is violated, L fails to find a feasible solution. However, one can readily see that

$$H_i = \frac{1}{3}T - 2\delta, \text{ for } i = 1, 2, 3,$$

is a feasible solution for the chosen C_i's, D_i's, and T. The feasibility of the above solution is verified in Appendix B.

C2: $h \le \frac{1}{3}T$: Let $h = \frac{1}{3}T - x$, where $x \ge 0$. We consider three subcases: $x > \frac{1}{6}T - \frac{\delta}{2}$, $x = \frac{1}{6}T - \frac{\delta}{2}$, and $0 \le x < \frac{1}{6}T - \frac{\delta}{2}$.
Subcase 1: $x > \frac{1}{6}T - \frac{\delta}{2}$. A again chooses

$$n = 3, C_2 = C_3 = C_1, \text{ and } D_2 = D_3 = D_1.$$

In this case, L will compute

$$H_i = h = \frac{1}{3}T - x < \frac{1}{6}T + \frac{\delta}{2} = \frac{C_i}{2},$$

for $i = 1, 2, 3$. Note that $q_i = 2$, for $i = 1, 2, 3$, and hence $X_i(\vec{H}) = H_i + \max(0, \min(r_i - 2H_i, H_i)) \le H_i + H_i < C_i$ since $\min(r_i - 2H_i, H_i) \le H_i$. Thus, the deadline constraint is violated and $\vec{H} = (h, h, h)$ is not a feasible solution for the chosen instance. However, as described in **C1**, a feasible solution, $H_i = \frac{1}{3}T - 2\delta$, for all i, indeed exists.
Subcase 2: $x = \frac{1}{6}T - \frac{\delta}{2}$, i.e., $H_1 = h = \frac{1}{3}T - x = \frac{1}{6}T + \frac{\delta}{2}$. In this case, A chooses

$$n = 2, C_2 = \frac{1}{2}T - \frac{3}{2}\delta + \epsilon, \text{ and } D_2 = 2T,$$

where $0 < \epsilon \le \frac{1}{6}T + \frac{\delta}{2}$. (The reason for choosing ϵ in the above interval is given in Appendix C).

Since $D_2 = 2T$ (i.e., $q_2 = 2$ and $r_2 = 0$), we have $X_2(\vec{H}) = H_2$. In order to meet the deadline constraint for the second message stream, L will compute

$$X_i(\vec{H}) = H_2 \ge C_2 = \frac{1}{2}T - \frac{3}{2}\delta + \epsilon.$$

Then, the minimum amount of time available for the first synchronous message stream becomes (note that $q_1 = 2$ and $r_1 = \frac{2}{3}T - \delta$)

$$\begin{aligned} X_1(\vec{H}) &= H_1 + \max(0, \min(r_1 - H_2, H_1)) \\ &\le \frac{1}{6}T + \frac{\delta}{2} + \max(0, \frac{1}{6}T + \frac{\delta}{2} - \epsilon) \\ &= \frac{1}{3}T + \delta - \epsilon < C_1, \end{aligned}$$

where the first inequality comes from $H_2 \ge \frac{1}{2}T - \frac{3}{2}\delta + \epsilon$, and the second equality comes from $\epsilon \le \frac{T}{6} + \frac{\delta}{2}$. From the above derivation, we conclude that the deadline constraint for the first message stream is not satisfied, and thus $\vec{H} = (\frac{1}{6}T + \frac{\delta}{2}, H_2)$ is not a feasible solution for the chosen instance. However, one can readily see that $\vec{H} = (H_1, H_2)$, where

$$\begin{aligned} H_1 &= \frac{1}{6}T + \frac{1}{2}\delta + \epsilon, \text{ and} \\ H_2 &= \frac{1}{2}T - \frac{3}{2}\delta + \epsilon = C_2 \end{aligned}$$

is a feasible solution, as is verified in Appendix C.
Subcase 3: $0 \le x < \frac{1}{6}T - \frac{\delta}{2}$. A chooses

$$\begin{aligned} n = 3, C_2 = C_3 = \frac{1}{3}T + \frac{x}{2} + \frac{\sigma}{2}, \text{ and} \\ D_2 = D_3 = 3T - C_2 = 3T - C_3, \end{aligned}$$

where $\sigma = (\frac{1}{6}T - \frac{\delta}{2}) - x > 0$. Since $C_2 = C_3$, $D_2 = D_3$, and L is a local SBA scheme, L will compute the same value for H_2 and H_3, i.e., $H_2 = H_3 = h'$. First, we show that $h' < C_2 = C_3$. If $h' \ge C_2 = C_3$, then $\sum_i H_i = h + h' + h' \ge T + \sigma > T$, violating the protocol constraint. This then implies (h, h', h') is not a feasible solution. We must have $h' < C_2 = C_3$. Under this restriction, it can be verified that $(\frac{1}{3}T - x, h', h')$, where $h' < C_2 = \frac{1}{3}T + \frac{x}{2} + \frac{\sigma}{2} = \frac{5}{12}T - \frac{\delta}{4}$ violates the deadline constraint for $i = 2, 3$ and is hence not a feasible solution. Specifically, we have $q_1 = 2$, $r_1 = \frac{2}{3}T - \delta$, $q_2 = q_3 = 2$, $r_2 = r_3 = \frac{7}{12}T + \frac{\delta}{4}$, and $X_2(\vec{H}) = H_2 + \max(0, \min(r_2 - H_1 - H_3, H_2))$. If $r_2 - H_1 - H_3 \le 0$, we have $\max(0, \min(r_2 - H_1 - H_3, H_2)) = 0$, and $X_2(\vec{H}) = H_2 = h' < C_2$. If $r_2 - H_1 - H_3 > 0$, we have $\max(0, \min(r_2 - H_1 - H_3, H_2)) \le r_2 - H_1 - H_3 = r_2 - H_1 - H_2$, and

$$\begin{aligned} X_2(\vec{H}) &\le H_2 + (r_2 - H_1 - H_2) \\ &= \frac{1}{4}T + \frac{\delta}{4} + x \\ &< \frac{5}{12}T - \frac{\delta}{4} = C_2, \end{aligned}$$

where the last inequality results from $0 \le x < \frac{T}{6} - \frac{\delta}{2}$. However, one can readily see that $\vec{H} = (H_1, H_2, H_3)$, where

$$\begin{aligned} H_1 &= \frac{1}{3}T - (\frac{1}{6}T - \frac{\delta}{2}) = \frac{1}{6}T + \frac{\delta}{2}, \text{ and} \\ H_2 &= H_3 = \frac{1}{4}T - \frac{3}{4}\delta, \end{aligned}$$

is a feasible solution, as is verified in Appendix D.

From **C1–2**, Theorem 4 follows. □

5 Conclusion

We formally proved that there does not exist any optimal local SBA scheme which is guaranteed to find a feasible solution for allocating synchronous bandwidths when such an allocation exists. This proof implies that the decision on choosing a non-optimal local scheme or an optimal global scheme depends on the trade-off between the ease of network management and the performance improvement.

Another contribution of this paper is an extension to the previous work on the bounded token rotation time. We prove that the time elapsed between the ℓ-th token departure from node b and the $(\ell + c)$-th token departure from node i is bounded by $(c+1) \cdot T + \sum_{j=b+1}^{i} H_j + \tau$ if $1 \le b < i \le n$, and is bounded by $c \cdot T + \sum_{j=b+1}^{i} H_j + \tau$ if $1 \le i \le b \le n$. The previous result on the upper bound of the token cycle time by Johnson *et al.* [5,6] and that on the upper bound of the time interval between c consecutive token's visits by Agrawal *et al.* [1,2] are special cases of our result.

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A Upper bound of δ

In both **C1** and **Subcase 1** of **C2** (Section 4), the parameters of the synchronous message streams are specified as: $C_1 = C_2 = C_3 = \frac{1}{3}T+\delta$, and $D_1 = D_2 = D_3 = 3T-(\frac{1}{3}T+\delta)$. And we claim that $H_1 = H_2 = H_3 = \frac{1}{3}T - 2\delta$ is a feasible solution. In this regard, we require that

- $H_i = \frac{1}{3}T - 2\delta > 0$, which leads to $\delta < \frac{1}{6}T$.
- $X_i(\vec{H}) \geq C_i$, where $X_i(\vec{H}) = (q_i - 1) \cdot H_i + \max(0, \min(r_i - 2H_i, H_i)) = H_i + \max(0, \min(r_i - 2H_i, H_i)) \leq 2H_i$, which leads to $2H_i \geq C_i$, or $2(\frac{1}{3}T - 2\delta) \geq \frac{1}{3}T + \delta$, i.e., $\delta \leq \frac{1}{15}T$.

Therefore, the range of δ is

$$0 < \delta \leq \frac{1}{15}T.$$

B Feasibility of the solution in C1

We now verify that the solution provided in **C1** and **Subcase 1** of **C2** in Section 4, i.e., $H_1 = H_2 = H_3 = \frac{1}{3}T - 2\delta$, is a feasible one (under the condition $0 < \delta \leq \frac{1}{15}T$) for the instance discussed in **C1** of Section 4: $C_i = \frac{1}{3}T + \delta$, and $D_i = 3T - C_i = 2T + (\frac{2}{3}T - \delta)$, for $i = 1,2,3$. Note that $q_i = 2$ and $r_i = \frac{2}{3}T - \delta$, for all i, for the above message stream configuration.

We need to verify that both the protocol and deadline constraints are satisfied. The protocol constraint is satisfied since $\sum_{i=1}^{3} H_i = T - 6\delta$. On the other hand, it suffices to check that $X_1(\vec{H}) \geq C_1$ for ensuring the deadline constraint to be met:

$$\begin{aligned} X_1(\vec{H}) &= (q_1 - 1) \cdot H_1 + \max(0, \min(r_1 - H_2 - H_3, H_1)) \\ &= \frac{1}{3}T - 2\delta + \max(0, \min(3\delta, \frac{1}{3}T - 2\delta)) \\ &= \frac{1}{3}T + \delta = C_1, \end{aligned}$$

where the third equality results from $\delta \leq \frac{1}{15}T$ (i.e., $\min(3\delta, \frac{1}{3}T - 2\delta) = 3\delta$).

C Feasibility of the solution in Subcase 2

We now verify that the solution provided in **Subcase 2** of **C2** in Section 4, i.e., $H_1 = \frac{1}{6}T + \frac{\delta}{2} + \epsilon$ and $H_2 = \frac{1}{2}T - \frac{3}{2}\delta + \epsilon$, where $0 < \epsilon \leq \frac{1}{6}T + \frac{\delta}{2}$, is a feasible one (under the condition $0 < \delta \leq \frac{1}{15}T$) for the instance discussed in that subcase: $C_1 = \frac{1}{3}T + \delta$, $C_2 = \frac{1}{2}T - \frac{3}{2}\delta + \epsilon$, $D_1 = 2T + (\frac{2}{3}T - \delta)$, and $D_2 = 2T$. Note that $q_1 = 2$, $r_1 = \frac{2}{3}T - \delta$, $q_2 = 2$, and $r_2 = 0$ for the above synchronous message configuration.

We first verify that the protocol constraint is satisfied: $H_1 + H_2 = \frac{2}{3}T - \delta + \epsilon \leq \frac{5T}{6} - \frac{\delta}{2} \leq T$, where the first inequality comes from $\epsilon \leq \frac{1}{6}T + \frac{\delta}{2}$. The deadline constraint is verified by calculating $X_1(\vec{H})$ and $X_2(\vec{H})$:

$$\begin{aligned} X_1(\vec{H}) &= (q_1 - 1) \cdot H_1 + \max(0, \min(r_1 - H_2, H_1)) \\ &= \frac{1}{6}T + \frac{\delta}{2} + \epsilon + \max(0, \frac{1}{6}T + \frac{\delta}{2} - \epsilon) \\ &= \frac{1}{3}T + \delta \;=\; C_1, \end{aligned}$$

where the third equality results from $\epsilon \leq \frac{T}{6} + \frac{\delta}{2}$, and

$$\begin{aligned} X_2(\vec{H}) &= (q_2 - 1) \cdot H_2 + \max(0, \min(r_2 - H_1, H_2)) \\ &= \frac{1}{2}T - \frac{3}{2}\delta + \epsilon + \max(0, \min(-H_1, H_2)) \\ &= \frac{1}{2}T - \frac{3}{2}\delta + \epsilon \;=\; C_2. \end{aligned}$$

D Feasibility of the solution in Subcase 3

We now verify that the solution provided in **Subcase 3** of **C2** in Section 4, i.e., $H_1 = \frac{1}{6}T + \frac{\delta}{2}$, $H_2 = \frac{1}{4}T - \frac{3}{4}\delta$, and $H_3 = \frac{1}{4}T - \frac{3}{4}\delta$, is a feasible one (under the condition $0 < \delta \leq \frac{1}{15}T$) for the instance discussed in that subcase: $C_1 = \frac{1}{3}T + \delta$, $D_1 = 2T + \frac{2}{3}T - \delta$, $C_2 = C_3 = \frac{1}{3}T + \frac{x}{2} + \frac{\sigma}{2} = \frac{5}{12}T - \frac{\delta}{4}$, and $D_2 = D_3 = 3T - C_2 = 3T - C_3 = 2T + (\frac{7}{12}T + \frac{\delta}{4})$, where $\sigma = (\frac{1}{6}T - \frac{\delta}{2}) - x > 0$. Note that $q_1 = 2$, $r_1 = \frac{2}{3}T - \delta$, $q_2 = q_3 = 2$, and $r_2 = r_3 = \frac{7}{12}T + \frac{\delta}{4}$ for the above synchronous message configuration.

The protocol constraint is satisfied since $H_1 + H_2 + H_3 = \frac{2}{3}T - \delta < T$. The deadline constraint is verified by calculating $X_1(\vec{H})$ and $X_2(\vec{H}) = X_3(\vec{H})$:

$$\begin{aligned} X_1(\vec{H}) &= (q_1 - 1) \cdot H_1 + \max(0, \min(r_1 - H_2 - H_3, H_1)) \\ &= \frac{1}{6}T + \frac{\delta}{2} + \max(0, \min(\frac{1}{6}T + \frac{\delta}{2}, \frac{1}{6}T + \frac{\delta}{2})) \\ &= \frac{1}{3}T + \delta = C_1, \end{aligned}$$

and

$$\begin{aligned} X_2(\vec{H}) &= (q_2 - 1) \cdot H_2 + \max(0, \min(r_2 - H_1 - H_3), H_2)) \\ &= \frac{1}{4}T - \frac{3}{4}\delta + \max(0, \min(\frac{1}{6}T + \frac{1}{2}\delta, \frac{1}{4}T - \frac{3}{4}\delta)) \\ &= \frac{5}{12}T - \frac{1}{4}\delta = C_2 = C_3, \end{aligned}$$

where the third equality comes from the fact that δ is in $(0, \frac{1}{15}T]$ (see Appendix A); therefore $\frac{1}{6}T + \frac{1}{2}\delta \leq \frac{1}{4}T - \frac{3}{4}\delta$.