Effects of Electromagnetic Interference on Controller-Computer Upsets and System Stability

Hagbae Kim, Allan L. White, and Kang G. Shin

***Abstract*—Electromagnetic interference (EMI) causes controller *upsets* manifested as errors in control-law computation on a controller computer, or as disturbances in data transmission between sensors/actuators and the controller computer, which may be critical to system stability. In this paper, we compute the probability of these EMI-induced upsets in terms of the parameters governing EMI behaviors and the conditional probabilities of upsets in the presence of EMI, which is useful for the design and verification of the integrity of reliable controllers. The results are used to modify a system dynamic equation to examine asymptotical stability *with probability one* for the random sequences of the system states evolving according to the resultant dynamic equation, which are validated via a simple EMI experiment in a reverberation chamber.**

***Index Terms*—Burst failure, controller upset, electromagnetic interference (EMI), stability, susceptibility.**

NOMENCLATURE

A, E, N, F	State of *EMI Absence*, *EMI Existence*, *No failure*, *Failure*.
λ_e and μ_e	Parameters of exponential distributions governing EMI occurrences and durations.
q and r	Transition probabilities $\Pr(A \rightarrow E)$ and $\Pr(E \rightarrow A)$.
p	Conditional probability of upset(s) given that EMI exists (in state E).
p_1 and p_2	Conditional probability of upset(s) in states N and F.
Π_f	Stationary probability of upset/failure occurrences.
Π_A, Π_N, Π_F	Stationary probabilities of states A, N, F.
j_1 and j_2	Total numbers of samples for the run lengths of N and F.
$\bar{n_1}$ and $\bar{n_2}$	Mean values of the samples for the run lengths of N and F.
$\mathbf{\Delta}_s, \mathbf{\Delta}_f, \mathbf{\Delta}_a$	Diagonal disturbance matrices (like $\mathbf{\Delta}$).
π and $E(\pi)$	Upset indicator and the mean of π, i.e., $E(\pi) = \Pi_F$.
$\mathbf{W}$ and $\mathbf{W}_a$	Desired system matrix with feedback control and the actual one.
$\lambda_{\max}$	Maximum eigenvalue of $(\mathbf{W}^T\mathbf{W} - \mathbf{I})$.

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I. INTRODUCTION

A DIGITAL computer has been increasingly used for monitoring and controlling real-time applications due to its high performance and reliability. It usually resides in the feedback loop of the system not only to periodically receive sensory data from the controlled plant but also to execute programmed tasks (e.g., control-law computation) using the data while correctly updating the control input at regular time intervals (T_s). This controller computer may be susceptible to an environmental disruption like electromagnetic interference (EMI) inducing external faults without damaging internal components. Unlike most internal faults that are permanent or intermittent due to physical defects (e.g., broken, short, or loose connections), external faults are likely to be transient and cause primarily functional error modes in a digital system, called *upsets* such as 1) changes in data values of the input–output (I/O) circuitry; 2) logic changes on the bus and control lines of the microprocessors; 3) logic changes in registers and/or ALU of the CPU, which consequently prevent the controller computer from generating correct control inputs.[1] The outcomes of these upsets induce computer failures such as missing the update of control inputs or generating erroneous control inputs during one or more sampling periods. Stationary occurrences of these upsets may lead to the loss of system stability either if their active duration exceeds a certain limit [3] or if they occur too frequently.

In [9], EMI effects on modern digital systems were reported to be substantial according to the answers to a questionnaire distributed to experts. In [1], a methodology for performing an upset test in a laboratory environment on a multichannel control system was presented with a case study of fault-tolerant electronic engine control systems, where EMI effects were primarily assessed but the work lacked analytic models or tools for interpreting the results. In [3] and [8], we previously analyzed the effects of the duration of computer failures on system stability by deriving the hard deadlines. However, the work could not capture direct effects of stationary occurrences of environmental disruptions on the control system (computer failure and thus system stability).

We thus investigate the effects of EMI on the controller computer as well as on the controlled plant at the same time. Specifically, we build a Markov-chain model for EMI behavior, and derive the stationary probability of controller upsets, which represents not only the level of *susceptibility* of the system to EMI, but also the occurrence rate of computer failures due to EMI. To analyze the direct effects of EMI on the controlled process, we modify the system dynamic equation by accounting for input disturbances caused by the upsets, and examine system

[1]We deal with the theoretic aspects of investigating the effects of the upsets, whereas only the first case of upsets on interfaces is verified through experiments.

stability. This information is invaluable to the design and verification of the integrity of reliable fault-tolerant controllers, because the probability of controller upsets can capture the relationships between the occurrence/recovery of computer failures and the level of fault-avoidance/fault-tolerance of the computers. Note that the occurrence of computer failure(s) induced by EMI depends on the electrical shielding and filtering properties of the structural materials against various intensities and frequencies of EMI, which prevent an exterior electromagnetic field from penetrating into the controller computer, as well as on the underlying error-/failure-handling scheme. The fact that EMI induces data changes in the digital circuitry is validated through laboratory experiments in a reverberation chamber coupling EM fields at various frequencies and voltages directly into data transmission lines (emulating the actuator or sensor lines), through which we estimate the parameters necessary to compute the stationary probability of controller upsets. We finally present an example of long-term control for the altitude of a spinning satellite showing the risk of frequent controller upsets, where the optimal control input may be corrupted by EMI and system stability may be lost if the stationary probability of controller upsets exceeds a certain limit.

II. Problem Statement

In such an adverse operating environment as high intensity radiated field (HIRF), EM field may cause an analog electrical signal/noise to be induced and propagated to on-board digital electronic systems, and some digital micro-electronic devices are more susceptible to these unwanted noises due to shrinking device size, lower switching energy, and higher speed operations. Thus, evaluating the susceptibility of digital controller-computers to EMI is essential to the development and verification of life-, safety-, and mission-critical systems like aircraft, nuclear reactors, and certain military systems. As mentioned earlier, the upsets due to EMI primarily result in temporary/transient functional error modes leading to control-law computation errors on the closed loop [1], [6]. Moreover, an EMI-rich harsh environment generally affects the entire system, regardless of the degree of redundancy used, thereby making the behaviors of external faults in different modules correlated with one another [2]. Unlike permanent faults caused by physical defects and manifested as stuck-at, bridging, or stuck-open faults, the faults resulting from EMI may not require any repair because there is usually no physical damage to the H/W. However, the computational tasks in the presence of these faults are contaminated and need to be recovered. Since a controller computer generally executes predefined control tasks with the sensed data and provides actuators with the control inputs periodically, once every T_s units of time, such transient faults can cause erroneous control inputs or input disturbances as a result of erroneous/no computations in the presence of EMI. (Note that in case the computer does not produce any output within T_s, the previous outputs may be used by the latch circuit.) It is also noteworthy that the computer sometimes operates normally in spite of the presence of EMI. Considering the above, we assume that all the effects of EMI on the controller are functional error modes, changing only some data values temporarily in control-law calculation (without any permanent damage) and that the controller returns normal in a certain time without special recovery action.[2] In addition, we assume that no upset occurs in the absence of EMI—although some internal faults due to manufacturing defects or wearing effects can still occur/persist, they are far from our concern to analyze the effects of EMI. EMI is generally characterized by a long latent period followed by a relatively short period of presence, whose occurrence rates were investigated in [7], [9]. However, for convenience we assume that EMI occurrences follow a time-invariant Poisson process with rate λ_e and its active duration has an exponential distribution with rate μ_e—the results can be modified for difference probability models of EMI behaviors if needed.

III. Effects of EMI on Stability

We now evaluate the controller's susceptibility to EMI by computing the stationary probability of controller upsets (computer failures) caused by EMI, from which one can obtain information about the rate of EMI causing upsets to controller computers. We then derive the maximum probability of controller upsets due to EMI to maintain asymptotic stability of the controlled plant.

A. Probability of Controller Upsets Induced by EMI

The EMI susceptibility model can be constructed in two parts. The first is to characterize the EMI behaviors using field data to be validated and estimated through EMI experiments, whereas the second is to build a model capturing the occurrences of controller upsets in the presence of EMI, whose parameters can also be measured experimentally. We specially focus on the second part and investigate the effects of controller upsets on the stability of the controlled plant. We begin with a simple model assuming that the probability of upset(s) during one time frame T_s in the presence of EMI is a constant, p (conditional probability of upset(s) given the occurrence of EMI), regardless whether the previous state is faulty or not—a more realistic model will be considered later covering bursty upsets in the presence of EMI. A Markov chain with two states is used to describe these aspects because each transition time is already assumed to have an exponential distribution. The two states will be called A (for *EMI Absence*) and E (for *EMI Existence*). In state A no upset occurs, whereas in state E an upset occurs with probability p like tossing a biased coin. The event in state E (upset or no upset) is not affected by the prior events (occurred during the previous time periods) at all. The Markov chain makes a transition depending on whether or not EMI occurs or disappears. The transition probabilities $q = \Pr(A \to E)$ and $r = \Pr(E \to A)$ are thus derived by the probability models of EMI behaviors assumed in Section II, that is,

$$q = \int_0^{T_s} \lambda_e e^{-\lambda_e t} dt = 1 - e^{-\lambda_e T_s} \quad \text{and}$$
$$r = \int_0^{T_s} \mu_e e^{-\mu_e t} dt = 1 - e^{-\mu_e T_s}. \tag{1}$$

[2]Without this assumption, we should consider methods for detecting faults/failures and providing safe recovery from permanent faults as well as quick recovery even from transient faults.

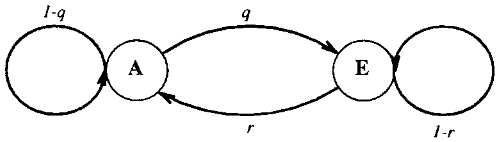

Fig. 1. A Markov-chain model representing EMI presence.

Fig. 1 is a transition diagram of the Markov chain, where runs of A will alternate with runs of E and the run lengths in a row have geometric distributions with mean $1/q$ for the A-runs and mean $1/r$ for the E-runs. (Geometric distributions under the assumption of independent events among different time frames are used for these runs due to its mathematical simplicity. However, one may consider more accurate models that are not only complicated but also useless without appropriate (vast) statistical data, which are difficult to collect.) Let Π_f be the stationary probability of upset occurrences. Since the stationary probability of state E, i.e., the fraction of time spent in E, is simply $\Pr(E) = q/(q+r)$ and an upset occurs only in state E with probability p, Π_f is equal to

$$\Pi_f = p\Pr(E) = \frac{pq}{q+r}. \quad (2)$$

We next consider a more realistic model generating a burst of controller upsets in the presence of EMI, for which state E should be classified into N (for *No upset/failure*) and F (for *Failure/upset*) depending on whether or not an upset occurs during the previous time frame. Before dealing with this model, we need to consider the event of EMI occurrences in detail, as depicted in Fig. 2(a). If an EMI occurs, the state moves from A to E. However, E in the case is a dummy state, whose holding time is zero, i.e., the state changes to either N or F instantaneously passing through E depending on whether or not an upset occurs due to EMI. Thus, state E is no longer necessary, and the diagram of Fig. 2(b) can capture all these phenomena. This Markov chain with three states can describe a burst of upsets in the presence of EMI. Instead of using p for the conditional probability of upset(s) in state E, two states N and F have different conditional probabilities of upset(s), defined by p_1 and p_2, respectively. State F must persist to simulate a burst of upsets with large p_2, which should also be much larger than p_1, because an upset is more likely to occur in an upset state during the previous time frame than in no upset state. In Fig. 2, $a = (1-r)p_1$ where $1-r$ is the probability that EMI is present and p_1 is the probability of an upset given that EMI is present. Similarly, $b = (1-r)(1-p_2)$. From this model, we can also derive the stationary probability of each state. Let Π_A, Π_N, and Π_F be the stationary probabilities (i.e., the fractions of time spent) of A, N, and F, respectively. Then, these probabilities $(\Pi_A + \Pi_N + \Pi_F = 1)$ are obtained by solving

$$\begin{aligned}\Pi_A &= (1-q)\Pi_A + r\Pi_N + r\Pi_F \\ \Pi_N &= q(1-p_1)\Pi_A + (1-r)(1-p_1)\Pi_N + (1-r)(1-p_2)\Pi_F \\ \Pi_F &= qp_1\Pi_A + p_1(1-r)\Pi_N + p_2(1-r)\Pi_F. \end{aligned} \quad (3)$$

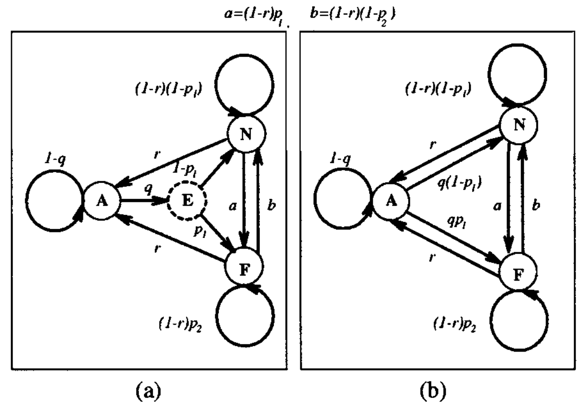

Fig. 2. Markov-chain models: (a) with an instant state E and (b) with three states.

In reality, $\Pr(E)$, the stationary probability of EMI presence, is equal to $\Pi_N + \Pi_F$

$$\Pi_N + \Pi_F = \frac{q}{q+r} \quad \text{and} \quad \Pi_A = \frac{r}{q+r}. \quad (4)$$

Hence, the probability of state F is computed by combining (3) and (4)

$$\Pi_F = qp_1\frac{r}{q+r} + p_1(1-r)\left(\frac{q}{q+r} - \Pi_F\right) + p_2(1-r)\Pi_F.$$

By solving the above equation for Π_F, we obtain

$$\Pi_F = \frac{q}{(q+r)}\frac{p_1}{[1-(1-r)(p_2-p_1)]}. \quad (5)$$

Consequently, we obtain the stationary probability of EMI-induced upset occurrences by (5), where q and r (equivalently, λ_e and μ_e) can be estimated using field data on EMI behaviors [9]. Although p_1 and p_2 are not directly observable, they can be deduced from statistical measurements using other easily measured parameters like the mean values of the samples for the run lengths of N and F. We designed an experiment emulating EMI and coupling EM fields directly into computer systems, where we observe the upset duration (measured in time frames) in the presence of EMI and counted the number of frames during which incorrect outputs are produced. This corresponds to the special case ($q = 1$ and $r = 0$) of the diagram of Fig. 2(b), implying that EMI always exists. We then collect the samples for the number of consecutive frames during which no upset occurs (runs of N in a row) and the numbers of consecutive frames during which upsets persist (runs of F in a row). For example, if we obtain the sets of the samples for both cases, we get $\{a_1, a_2, \ldots, a_{j_1}\}$ and $\{b_1, b_2, \cdots, b_{j_2}\}$, where j_1 are j_2 are the total numbers of samples for both cases. When $s_1 = \sum_{i=1}^{j_1} a_i$ and $s_2 = \sum_{i=1}^{j_2} b_i$ are defined as the sums of these samples, the total number of frames becomes $s_1 + s_2$. Let $\bar{n_1}$ and $\bar{n_2}$ be the mean values of the samples for the run lengths of N and F, respectively, then $\bar{n_1} = s_1/j_1$ and $\bar{n_2} = s_2/j_2$. Also, we analytically derive these parameters from the case of $q = 1$ and $r = 0$ in the diagram of Fig. 2(b). The mean length of F-runs is

$$\begin{aligned} E(n_2) &= \sum_{k=1}^{\infty} k p_2^{k-1}(1-p_2) \\ &= (1-p_2)\sum_{k=1}^{\infty} k p_2^{k-1} = (1-p_2)\sum_{k=1}^{\infty} \left(p_2^k\right)' \\ &= (1-p_2)\left(\sum_{k=1}^{\infty} p_2^k\right)' = (1-p_2)\left(-1+\sum_{k=0}^{\infty} p_2^k\right)' \\ &= (1-p_2)\left(-1+\frac{1}{1-p_2}\right)' = \frac{1}{1-p_2}. \end{aligned} \tag{6}$$

From $\bar{n}_1 = E(n_1)$ and $\bar{n}_2 = E(n_2)$, we can obtain the estimators (denoted by $\bar{p_1}$ and $\bar{p_2}$) for p_1 and p_2 in terms of $\bar{n}_1$ and $\bar{n}_2$

$$\begin{aligned} \bar{n}_2 &= \frac{1}{1-\bar{p}_2} \Rightarrow \bar{p}_2 = 1 - \frac{1}{\bar{n}_2} = 1 - \frac{j_2}{s_2} \\ \bar{n}_1 &= \frac{1}{\bar{p}_1} \Rightarrow \bar{p}_1 = \frac{1}{\bar{n}_1} = \frac{j_1}{s_1}. \end{aligned} \tag{7}$$

B. Features and Effects of Upsets on System Stability

A controller computer calculates the control input at each sampling period for a linear time-invariant controlled process that is described by the vector difference equation

$$\mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k); \quad \mathbf{x}(k_0) = \mathbf{x}_0 \tag{8}$$

where k indicates each time frame (e.g., T_s) measured from an initial time k_0. Here, $\mathbf{x}(k) \in \mathcal{R}^n$ is the system state, $\mathbf{u}(k) \in \mathcal{R}^m$ the applied control, and the matrices $\mathbf{A}$ and $\mathbf{B}$ are bounded, which can be obtained from the corresponding continuous-time model [8]. We start our analysis by formally defining system stability in terms of the concept of exponential convergence of a sequence $\mathbf{x}[k_0, \ldots]$. That is, $\mathbf{x}(\cdot)$ converges exponentially to zero if there exist positive constants α and M such that $|\mathbf{x}(k)| < M\exp[-\alpha(k-k_0)]$ for all $k > k_0$, where $|\cdot|$ is a vector norm. The system is defined as "stable-in-the-mean" if $E[\mathbf{x}(k)]$, which is the mean of the state at the kth time frame, converges exponentially to a certain equilibrium state of the system (say $\mathbf{y}$). Also, the system is defined as asymptotically stable with probability one if, for each $\epsilon > 0, |\mathbf{x}(k) - \mathbf{y}| \geq \epsilon$ only finitely often, that is, the random sequence $\mathbf{x}(k)$ converges to $\mathbf{y}$ for all sets of events except possibly on a set of events having probability zero. Note that some recent and future systems such as highly fuel-efficient "fly-by-wire" aircraft demand the reliable control actions to maintain stability (*stabilizable*) because those are likely to be intrinsically unstable to achieve other purposes or go through the edge of stability. Thus, frequent upsets of the controller induced by EMI may affect more seriously system stability in those systems. We specifically attend to a system using a certain feedback control input, $\mathbf{u}(k) = \mathbf{F}\mathbf{x}(k)$, for stabilizing the system matrix $\mathbf{A}$ while optimizing a certain performance index. As described earlier, during an upset the controller computer fails to provide the physical actuator with correct control inputs due to either 1) control-law calculation errors resulting from logic changes inside processors or 2) transmission disturbances caused by data changes on sensor/actuator lines or on I/O circuitry containing the interfaces such as A/D and D/A converters. In other words, the control input during an upset becomes $\mathbf{u} + \mathbf{\Delta}_0 = (\mathbf{I} + \mathbf{\Delta})\mathbf{u}$, where $\mathbf{I}, \mathbf{\Delta}_0$, and $\mathbf{\Delta}$ are an identity matrix, a vector ($\in \mathcal{R}^m$), and a diagonal matrix with random-sequence elements, $\mathrm{diag}[\mathbf{\Delta}]_i = \Delta_i$, modeled by the bounded outputs of certain dynamic systems with white-noise sequences, respectively. Let $\mathbf{u}_a(k)$ be an actual control input, which becomes a desirable one $\mathbf{F}\mathbf{x}(k)$ or a disturbed one, depending on whether or not an upset occurs. The disturbed one can also be described in detail as follows:

- in case of control-law calculation errors, $\mathbf{u}_a = (\mathbf{I} + \mathbf{\Delta}_f)\mathbf{F}\mathbf{x}$;
- in case of transmission errors on the sensor line, $\mathbf{u}_a = \mathbf{F}(\mathbf{I} + \mathbf{\Delta}_s)\mathbf{x}$;
- in case of transmission errors on the actuator line, $\mathbf{u}_a = (\mathbf{I} + \mathbf{\Delta}_a)\mathbf{F}\mathbf{x}$;

where $\{\mathbf{\Delta}_s, \mathbf{\Delta}_f, \mathbf{\Delta}_a\}$ are diagonal disturbance matrices (like $\mathbf{\Delta}$) which are independent of one another. The mean and the variance of each disturbance matrix (random sequence) is given as μ_a and σ_a for $\mathbf{\Delta}_s$, μ_f and σ_f for $\mathbf{\Delta}_f$, and μ_a and σ_a for $\mathbf{\Delta}_a$, measurable through experiments or modeled by the outputs of dynamic systems with white-noise inputs. To modify the system dynamic equation (8) accounting for the upset effects, we represent $\mathbf{u}_a(k)$ affected by an upset as

$$\mathbf{u}_a(k) = (\mathbf{I} + \mathbf{\Delta}_a)(\mathbf{I} + \mathbf{\Delta}_f)\mathbf{F}(\mathbf{I} + \mathbf{\Delta}_s)\mathbf{x}(k) \tag{9}$$

which covers all three features described above. If the system has an ideal scheme detecting failures perfectly and instantaneously, one can suggest better control strategies to hold the control inputs at the previous values or to set $\mathbf{u}_a(k) = \mathbf{0}$ (i.e., $\mathbf{\Delta} = -\mathbf{I}$) during the computer failures, which was used to derive the control system deadline in the presence of computer failures and perfect detection coverage in [8]. However, we consider general and practical cases of random (arbitrary) control inputs generated as a result of controller-computer failures. Let π be an upset indicator, which will be zero or one depending on the occurrence of an upset during each time frame, then the mean of π is equal to Π_F as derived before, i.e., $E(\pi) = \Pi_F$. By using this sequence of binary upset digits, we can rewrite the system dynamic equation as follows:

$$\begin{aligned} \mathbf{x}(k+1) &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}_a(k) \\ &= \mathbf{A}\mathbf{x}(k) + \mathbf{B}(1-\pi)\mathbf{F}\mathbf{x}(k) \\ &\quad + \pi\mathbf{B}(\mathbf{I} + \mathbf{\Delta}_a)(\mathbf{I} + \mathbf{\Delta}_f)\mathbf{F}(\mathbf{I} + \mathbf{\Delta}_s)\mathbf{x}(k) \\ &= [\mathbf{A} + \mathbf{B}(1-\pi)\mathbf{F} + \pi\mathbf{B}(\mathbf{I} + \mathbf{\Delta}_a) \\ &\quad \times (\mathbf{I} + \mathbf{\Delta}_f)\mathbf{F}(\mathbf{I} + \mathbf{\Delta}_s)]\mathbf{x}(k). \end{aligned} \tag{10}$$

As a result of stationary-occurring controller upsets, the system matrix $\mathbf{W} = \mathbf{A} + \mathbf{B}\mathbf{F}$, which was changed from $\mathbf{A}$ to make the system stable while minimizing a certain cost index (e.g., optimizing a quadratic performance index given by (14)), is now changed to a *random matrix*

$$\mathbf{W}_a = \mathbf{A} + \mathbf{B}(1-\pi)\mathbf{F} + \pi\mathbf{B}(\mathbf{I} + \mathbf{\Delta}_a)(\mathbf{I} + \mathbf{\Delta}_f)\mathbf{F}(\mathbf{I} + \mathbf{\Delta}_s). \tag{11}$$

1) Test of Stability-in-the-Mean: Using this system matrix to account for the effects of EMI as well as controller upsets, we examine the system stability. As a simplest case, we compute the mean of the system matrix from this stochastic equation (11)

$$E[\mathbf{W}_a] = \mathbf{A} + \mathbf{B}(1 - \Pi_F)\mathbf{F} + \Pi_F \mathbf{B}(\mathbf{I} + \mu_a)(\mathbf{I} + \mu_f)\mathbf{F}(\mathbf{I} + \mu_s). \quad (12)$$

This computation clearly depends on the probability model of disturbances $\{\mathbf{\Delta}_f, \mathbf{\Delta}_s, \mathbf{\Delta}_a\}$, which are assumed to be independent of one another. Although one can simply examine the stability condition for the mean of state trajectories, it has little physical sense because there may be some sample trajectories (events) losing system stability in spite of meeting the stability condition of (12), that is, having all eigenvalues smaller than one (inside the unit circle).

2) Test of Asymptotical Stability with Probability One: We now develop a method to examine a stronger condition guaranteeing asymptotic stability—associated to (infinitely long) trajectory of the system composed of all possible sequences of matrices—with probability one (*w.p. 1*). The probability distribution of the matrices induces a (rather complicated) probability distribution on the trajectory space. Thus, the results of the paper handle the properties about the trajectory space in terms of parameters describing the distribution of the matrices. One property of random variables is that if X_n is a random sequence and $\sum E[|X_n|] < \infty$, then $X_n \to 0$ *w.p. 1*, which is treated in the Borel-Cantelli lemma [4]. This gives conditions for stability in terms of the first moments of a scalar function, the matrix norm ($\|\cdot\|$). The classical result for deterministic systems is that if the norm is less than one then the system is geometrically asymptotically stable, whereas the stochastic result is represented by the following theorem.

Theorem 1: If $E[\|W\|] < 1$, 1) the system is asymptotically stable *w.p. 1*; 2) $E[|x(k)|]$ converges to zero geometrically. If $E[\|W\|^2] < 1$; and 3) the system is geometrically asymptotically stable in the mean square sense.

Proof: By the strong law of large numbers, $\sum_{i=0}^{n} \|\mathbf{W}_i\|/n$ converges to the average *w.p. 1*. Since the geometric mean is less than or equal to the arithmetic mean

$$\|\mathbf{W}_n \cdots \mathbf{W}_1 v\| \le \|\mathbf{W}_n\| \| \cdots \|\mathbf{W}_1\| \le [(\|\mathbf{W}_n\| + \cdots + \|\mathbf{W}_1\|)/n]^n.$$

The right-hand side goes to zero *w.p. 1* as $n \to \infty$, proving part 1). The proofs of 2) and 3) are straightforward. □

In general, neither convergence *w.p. 1* nor convergence in mean square implies each other. For the above criterion, the condition for convergence in mean square implies the condition for convergence *w.p. 1* because of the Cauchy–Schwartz inequality, $(E[|X|])^2 \le E[|X|^2]$. Next we consider an eigenvalue condition for the sufficient condition of stability *w.p. 1*. To this end, we present the following lemma about the exponential stability of random sequences (see [4] for proof). Let $V(x)$ be a Lyapunov function, which is a nonnegative real-valued function with continuous derivatives and $V(0) = 0$, such that $V(x) \to \infty$ as $|x| \to \infty$.

Lemma 1: If $V(\mathbf{x}(k)) \ge 0$ for all k, and $E[V(\mathbf{x}(1))] - V(\mathbf{x}) \le -\alpha V(\mathbf{x})$ for some $\alpha > 0$, then $E[V(\mathbf{x}(k))] \le (1-\alpha)^k V(\mathbf{x})$ and $V(\mathbf{x}(k)) \to 0$ *w.p. 1*.

Based on this lemma, we propose the following theorem.

Theorem 2: Let the state $\mathbf{x}$ evolve according to a system matrix $\mathbf{W}$, which is a random matrix having some random sequences as its elements. If all eigenvalues of $E[\mathbf{W}^T\mathbf{W}] - \mathbf{I}$ are smaller than zero, then $|\mathbf{x}(k)| \to 0$ *w.p. 1*.

Proof: Let $V(\mathbf{x}) = \mathbf{x}^T\mathbf{x}$. Recall $\mathbf{x}(k+1) = \mathbf{W}\mathbf{x}(k)$, and consider Lemma 1

$$E[V(\mathbf{x}(1))] - V(\mathbf{x}) = E[(\mathbf{W}\mathbf{x})^T(\mathbf{W}\mathbf{x})] - \mathbf{x}^T\mathbf{x} = \mathbf{x}^T E[\mathbf{W}^T\mathbf{W} - \mathbf{I}]\mathbf{x}.$$

Since $(\mathbf{W}^T\mathbf{W} - \mathbf{I})$ is symmetric, $E[V(\mathbf{x}(1))] - V(\mathbf{x}) \le \lambda_{\max}\mathbf{x}^T\mathbf{x}$, where $\lambda_{\max}$ is the maximum eigenvalue of $(\mathbf{W}^T\mathbf{W} - \mathbf{I})$. Hence, if all eigenvalues of $E[\mathbf{W}^T\mathbf{W}] - \mathbf{I}$ are smaller than zero, by Lemma 1 (by assigning $\alpha = -\lambda_{\max} > 0$), $\mathbf{x}(k)^T\mathbf{x}(k) \to 0$ *w.p. 1*. Therefore, $|\mathbf{x}(k)| \to 0$ *w.p. 1*. □

Let $\mathrm{eig}(\mathbf{A})$ be an eigenvalue of $\mathbf{A}$, then we have the following proposition.

Proposition 1: Since a symmetric matrix can be diagonalized by an orthogonal transformation

$$\mathrm{eig}(E[\mathbf{W}^T\mathbf{W}] - I) < 0 \Leftrightarrow \mathrm{eig}(E[\mathbf{W}^T\mathbf{W}]) < 1.$$

Consequently, we can examine asymptotic stability *w.p. 1* by obtaining the eigenvalues of $E[\mathbf{W}^T\mathbf{W}]$, where $\mathbf{W}$ is the random matrix of (11) modified to account for the EMI effects. When there is some correlation between the disturbances $\{\mathbf{\Delta}_f, \mathbf{\Delta}_s, \mathbf{\Delta}_a\}$, the computation of $E[\mathbf{W}^T\mathbf{W}]$ becomes more complicated due to nonzero covariances of the disturbances. However, if we assume independence among the disturbances like (12), this computation needs only the variance of each disturbance and its mean value. Applying these results, we can derive the maximum value of Π_F maintaining system stability for certain disturbances with given mean and variance. Intuitively, we expect that as the probability of upset occurrences increases the stability property is less likely to be preserved by comparing the maximum eigenvalues for various values of Π_F. From the requirements of Π_F, we can consequently obtain the conditions of p_1 and p_2, which are determined by the electrical shielding properties against various intensities and frequencies of EMI.

IV. Experiments and an Example

The EMI experiment aims at emulating the physical effects of EMI on the digital systems and determining how the EMI corrupted the signals. Since we analytically derived the mean of the run lengths for states N and F using the necessary parameters and the property of a geometric distribution, the sample mean obtained through this EMI experiment will enable us to estimate the necessary parameters using (7). This procedure corresponds to the second part of constructing the EMI susceptibility model, i.e., to model the occurrences of controller upsets in the presence of certain EMI. In the experiment, EMI affecting only transmission lines and (avionics) interfaces is considered, where all

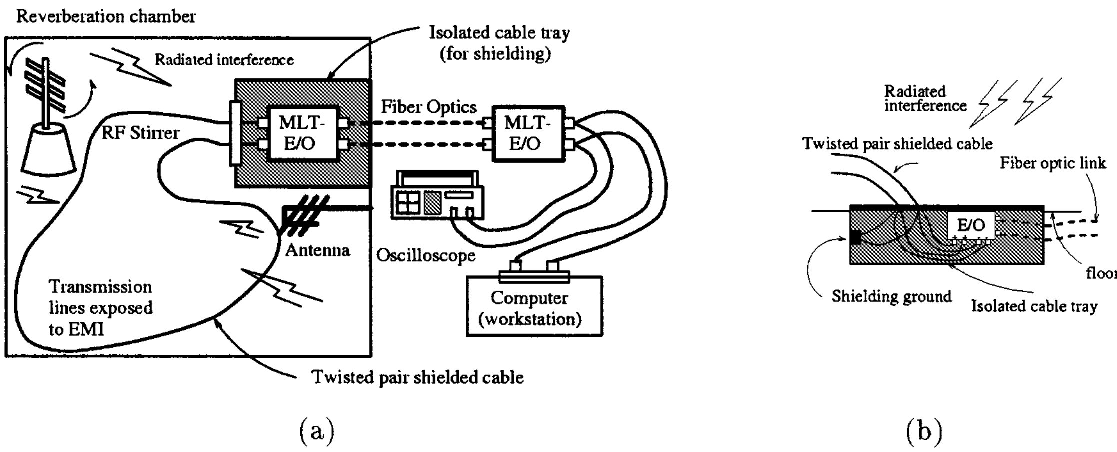

Fig. 3. The experimental setup: (a) signal transmission test set-up; (b) inside the cable tray.

other experiment equipments like an electric/optic (E/O) converter should be completely shielded against EMI, examined via pre-experimental tests. Specifically, the experiment is designed to generate upsets on the communication loop due to noise induced by the radiating radio frequency (RF) field in the reverberation chamber, as depicted in Fig. 3. Although there are two modes of EMI coupling, radiated, and conducted, E/O converters can avoid propagating EMI effects outside the chamber through the conducted mode of coupling. (Note that the conducted path may be resistive, or some inductance and capacitance, or combination thereof.) The multilink transceiver (MLT) is placed within an isolated cable tray located under the floor of chamber. Eighteen feet of twisted pair shielded cable[3] is connected to the MLT transmitter terminal, draped in free-space within the HIRF chamber, and returned to the receive terminal connection. The cable shields are connected to an earth ground, as shown in Fig. 3, inside of the tray with an EM-field sensor. Once these devices are secure, the chamber is closed and the stirrer rotates with a certain speed (e.g., 5 r/min) and the signal having a certain frequency and power level, which are controllable, is radiated from the antenna into the chamber. An oscilloscope is also connected to the transmit and receive terminal strip, of the up-stream MLT to observe any false signal transmission in the communication loop. Four-byte streams were sent through the chamber under EMI having stirred- and continuous-wave power.

- Stream 1: 2 K, each of a *uniform* distribution, *normal* distribution, a *sawtooth*, and a *ramp*.
- Stream 2: 8 K of *ramp* from 0 to 255
- Stream 3: 2 K, each of *ramp*, *sawtooth*, *normal* distribution, and *uniform* distribution.
- Stream 4: 8 K of *ramp* from 255 to 0.

At each frequency and power, streams 1 and 2 were each sent five times. If there was no disturbance, the next frequency or power level was selected. If there was a disturbance, streams 3 and 4 were each sent five times. There was no upset on the transmission under the frequencies 475, 500, and 575 MHz, but we observed some upsets under 525 and 550 MHz. Table I shows the estimated p_1 and p_2 in the presence of EMI having these two frequencies for various power levels, to which transmission upsets are most sensitive. We expect that there are critical frequencies[4] of EMI contributing more significantly to upsets, depending upon the frequency of the carrier signal and the type/geometry of placement of the transmission lines in the chamber. The results showing $(1-p_1) \gg p_1$ and $p_2 \gg (1-p_2)$ support our model covering bursty upsets. Suppose that the parameters of EMI behaviors are given by $\lambda_e T_s = 9.26 \times 10^5$ and $\mu_e T_s = 0.033$. For example, if T_s is one second, the mean rate of EMI occurrences is 3 h and the mean duration is 30 s. We then estimate Π_F for various frequencies and power levels, as given in Table I. If the transmission lines used in this experiment are assumed as the sensor or the actuator lines, the effects of EMI on system stability or performance can be analyzed using the estimated Π_F.

TABLE I
$(\bar{n}_1, \bar{n}_2)$ OBTAINED BY THE EXPERIMENT, $(\bar{p}_1, \bar{p}_2)$ AND Π_F DERIVED BY EQS. (3.7) AND (3.5), RESPECTIVELY

Freq $[Mh]$: Power $[watts]$	$(\bar{n}_1, \bar{n}_2)$	$(\bar{p}_1, \bar{p}_2)$	Π_F
525 : 2	(7375, 12)	(0.000136,0.917)	3.367×10^{-6}
525 : 5	(6966, 18)	(0.000144,0.945)	4.691×10^{-6}
525 : 10	(5340, 16)	(0.000187,0.938)	5.678×10^{-6}
525 : 50	(16, 14)	(0.000608,0.928)	1.663×10^{-5}
550 : 2	(6149, 13)	(0.000163,0.925)	4.347×10^{-6}
550 : 5	(6972, 10)	(0.000143,0.904)	3.210×10^{-6}
550 : 10	(5988, 7)	(0.000167,0.861)	2.807×10^{-6}
550 : 50	(1232, 25)	(0.000812,0.960)	3.160×10^{-5}

We now present a simple example[5] to determine the stationary probability of controller upsets (Π_F) leading to loss of asymptotical stability *w.p. 1* as well as stability-in-the-mean. The dynamic behavior of the altitude of a spinning satellite is

[3]The transmission type is RS422 with 12.5 Kbytes per second. There is a check to see if anything is received and a parity check. If nothing was received or if an incorrect result was detected (by parity), then the controller gave the zero (or neutral response) as if no corrective action was needed.

[4]Dividing the speed of electromagnetic wave (3×10^8 m/s) by the length of the wire (5.54 m) gives 5.4×10^7 s. Hence, 540 MHz is a multiple of the resonant frequency of the wire. We believe this accounts for being able to disturb the system at very low power levels of 525 and 550 MHz.

[5]Extracted from [5] but with modified coefficients.

TABLE II

$\lambda_{\max}(E[\mathbf{W}^T\mathbf{W}])$ FOR VARIOUS VALUES OF THE FIRST AND SECOND MOMENTS OF $\mathbf{\Delta}_a$ AND $\mathbf{\Delta}_f$ WHILE VARYING Π_F; *CASE 1:* $\{-2\mathbf{I}, 2\mathbf{I}, 20\mathbf{I}, 20\mathbf{I}\}$, *CASE 2-a:* $\{-5\mathbf{I}, 5\mathbf{I}, 20\mathbf{I}, 20\mathbf{I}\}$, *CASE 2-b:* $\{-5\mathbf{I}, 5\mathbf{I}, 50\mathbf{I}, 50\mathbf{I}\}$, *CASE 2-c:* $\{-5\mathbf{I}, 5\mathbf{I}, 100\mathbf{I}, 100\mathbf{I}\}$, *CASE 3:* $\{-10\mathbf{I}, 10\mathbf{I}, 100\mathbf{I}, 100\mathbf{I}\}$, FOR $\{\mu_a, \mu_s, \sigma_a, \sigma_s\}$ AND $\mathbf{\Delta}_f = \mathbf{0}$, RESPECTIVELY

Π_F	*case 1*	*case 2-a*	*case 2-b*	*case 2-c*	*case 3*
1×10^{-8}	0.5327	0.5337	0.5362	0.5430	0.5595
5×10^{-8}	0.5344	0.5393	0.5517	0.5859	0.6687
1×10^{-7}	0.5364	0.5463	0.5711	0.6396	0.8059
5×10^{-7}	0.5531	0.6023	0.7269	1.0724	1.9099
1×10^{-6}	0.5740	0.6726	0.9226	1.6159	3.2935
5×10^{-6}	0.7417	1.2385	2.4968	5.9741	14.3702
2×10^{-5}	1.3752	3.3712	8.4123	22.3257	55.9119

described in terms of the long-term control of the roll (φ) and yaw (ψ) angles, which is based on the dynamic coupling resulting from the rotation of the satellite around the earth

$$\begin{aligned}\dot{\varphi}_x &= 2.1\varphi_x + 1.5\psi_z + 0.1u_x + 0.2u_z\\ \dot{\psi}_z &= -2.3\psi_z + 0.2u_x + 0.1u_z\end{aligned} \tag{13}$$

where the coefficients depend upon the orbital frequency, i.e., the angular velocity of the satellite with respect to the inertial frame, and u_x and u_z are control signals. The goal of the control is to maintain a desired orientation of the satellite in the orbit around the earth (the stabilization problem) with the minimum-control effort, which results in the optimal (feedback) control gain matrix $\mathbf{F}$ by minimizing a quadratic performance index

$$J = \frac{1}{2}\left(\sum_{k=0}^{k_f-1}\left[\mathbf{x}^T(k)\mathbf{Q}(k) + \mathbf{u}^T(k)\mathbf{R}(k)\right] + \mathbf{x}^T(k_f)\mathbf{Q}(k_f)\right). \tag{14}$$

Suppose that $\mathbf{Q} = 2\mathbf{I}$ and $\mathbf{R} = 5\mathbf{I}$ are determined by the control objective of interest and $T_s = 1$. The corresponding coefficient matrices are then

$$\mathbf{A} = \begin{bmatrix} 8.1660 & 2.7490\\ 0 & 0.1003\end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0.5470 & 0.7855\\ 0.0782 & 0.0391\end{bmatrix};$$
$$\mathbf{F} = \begin{bmatrix} 4.7619 & 1.6244\\ 6.6524 & 2.2661\end{bmatrix}.$$

This feedback control changes the eigenvalues from $\{8.166, 0.1003\}$ to $\{0.1102 \pm 0.0045j\}$, thus stabilizing the satellite. When we introduce some EMI effects generating upsets in the controller, the eigenvalues are changed by varying the frequency in stationary occurrences of upsets. In Table II, the maximum eigenvalues of $E[\mathbf{W}^T\mathbf{W}]$ are shown for various values of the mean and the variance of each disturbance (assumed to occur only on the sensor and/or the actuator lines), while varying Π_F. Using Theorem 2 and these values, we can examine asymptotic stability *w.p. 1* in the usual sense of stability properties for random sequences. We see that the stability property is less preserved as Π_F increases. When Π_F becomes equal to or greater than a certain threshold probability, which is 1.113×10^{-5} for *case 1*, 3.32×10^{-6} for *case 2-a*, 1.12×10^{-6} for *case 2-b*, 4.34×10^{-6} for *case 2-c*, and 1.8×10^{-7} for *case 3*, the system has at least one eigenvalue greater than or equal to one, implying the loss of system stability in the sense of asymptotical stability with *w.p. 1*. Obviously from case 2-$\{a, b, c\}$, the system becomes unstable with a larger variance of disturbance, implying that the occurrences of upsets should be kept below a smaller rate to maintain system stability when information about disturbance magnitudes is more uncertain. As a result, one should design the controller sufficiently tolerant of EMI to have the corresponding p_1 and p_2, or equivalently to meet the condition $\Pi_F < \Pi_0$ thus retaining system stability.

V. CONCLUSION

We investigated the effects of EMI both on the controller computer and on the control plant simultaneously. First, we derived the stationary probability of upsets induced due to EMI, which represents the level of susceptibility of the controller to EMI depending upon the EMI behaviors and the shielding properties of the materials and structures of the controller against EMI. We also examined system stability by using a stochastic dynamic equation modified to account for the effects of upsets. The results showed the effects on EMI on the control plant indirectly through the probability of upsets in the controller. We presented an example examining system stability and the EMI experiment, in a HIRF chamber that was constructed at the NASA Langley Research Center, estimating the necessary parameters to demonstrate our theoretical work.

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