# Performance Analysis of the Interactivity for Multicast True VoD Service

Huadong Ma[1] and Kang G. Shin[2]
1.College of Computer Sci.&Tech., Beijing University of Posts and Telecommunications, Beijing 100876, China
2.RTCL, EECS Dept., The University of Michigan, Ann Arbor, MI48109-2122, USA

*Abstract*—**In this paper, we propose a new approach for analyzing the user interactivity and evaluating the number of channels required for multicast TVoD service. Our results determine the relationships among the clients' behaviors, the system resources in server and customer promise equipment, and the TVoD service protocol. Extensive simulation results support the validity/reasonableness of our analysis results.**

## I. INTRODUCTION

A VoD service allows remote clients to play back any video from a large collection of videos, stored at one or more video servers at any time. VoD service is usually long-lived and real-time, requires high storage-I/O & network bandwidths, and supports VCR-like interactivity. VoD clients' QoS is related to service latency, interactivity, and playback effects. Usually, there is a trade-off between the clients' QoS and system cost. In this paper, a *server channel* is defined as the resource required to support one video stream while guaranteeing continuous playback for a client.

TVoD service supports all of control functions such as Play (PLY)/Resume (RS), Stop (STP)/Pause (PAU)/Abort (ABT), Fast Forward (FF)/Rewind (REW), Fast Search (FS)/Reverse Search (RS), Slow Motion (SM). The conventional TVoD system uses one dedicated channel for each service request, which offers the client the best QoS and TVoD service. However, it incurs high system costs, especially in terms of storage-I/O and network bandwidth. One efficient solution to these problems is to use multicast. Multicast VoD system has excellent scalability and cost/performance efficiency.

We want multicast VoD systems to support VCR-like interactivity while improving service efficiency. There have been approaches to solving this problem [1], [2], [6], but there have been very few attempts to quantify the bandwidth requirement of multicast TVoD service and evaluate the overall storage-I/O and network-I/O throughputs. In this paper, we focus on the performance analysis of the interactivity for multicast TVoD protocols, and propose a new approach to analyzing the user activity of multicast TVoD service, and evaluating the channel requirement for TVoD service. These results formally specify the relationships among the clients' behavior, the system resource and the TVoD protocol.

## II. MULTICAST TVOD APPROACHES

Some efficient techniques have been proposed for multicast VoD interactions. The authors of [3] introduced *tuning* in staggered VoD which broadcasts multiple copies of the same video at staggered times. Intelligent tuning to different broadcast channels is used to achieve a Near VoD (NVoD) interaction. Other initial approaches to the VCR functionality are proposed in [2] and [4] for handling pause/resume requests. The support for continuous service of pause operations was simulated for NVoD service, but merge operations from an interaction (I-) channel to a multicast channel were either ignored [4], or did not guarantee continuity in video playout [2]. The authors of [2] proposed use of the CPE buffer for limited interactions.

In order to realize TVoD interactions in multicast VoD service, the SAM protocol [6] offers an efficient way, and all the interactions are served by allocating interaction (I-) streams as soon as a VCR action request is issued. When playout resumes, the VoD server attempts to merge the users back into a multicast service (S-) stream by using a dedicated *synchronization buffer* located at the server or access nodes and partially shared by all the users. Should this attempt fail, a request for a new S-stream is then initiated. The authors of [1] improved the SAM protocol by using the CPE buffer and active buffer management, thus allowing more interactions to be served without requiring additional I-channels. After a user finishes an interaction, his I-stream should be merged into a regular multicast service stream, as shown in Figure 1. Let $A$ be the desired play point or target point, and $P(n)$ be the play point of the nearest stream that the I-stream can be merged into. Then, the I-channel should be extended for the time period of $a = |P(n) - A|$, and the skew between $A$ and $P(n)$ is absorbed in merging.

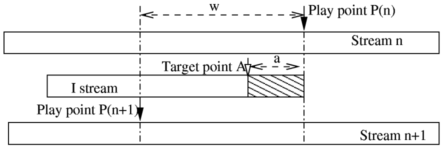

Figure 1. Merging

The SAM protocol [6] uses the synchronization buffer to merge the I- and S- streams. Each user resuming normal play after an interaction operation requires a video stream, but no such real stream may exist, so SAM attempts to create a "virtual" stream by using an ongoing S-stream and the synchronization buffer. The protocol identifies the closest ongoing S-stream — the one with the smallest offset in time from the virtual stream. This real S-stream feeds the synchronization buffer, and the virtual stream retrieves content from the synchronization buffer after the required time offset. The improved SAM protocol in [1] uses the CPE buffer to merge the I- and S- streams instead of using the synchronization buffer, thus reducing the server and access node's overheads.

In summary, multicast TVoD service is composed of the following three phases:

The work reported in this paper is supported by the National Science Foundation of China under Grant 69873006 and the US NSF under Grant EIA-9806280. This work was partly done when Ma visited EECS Dept.,University of Michigan.

*Admission*: The objective of this phase is to admit clients' requests without delay. To this end, there have been several dynamic multicast schemes proposed that allow an existing multicast to expand dynamically and serve new clients at zero-delay, such as Patching [5].

*Interaction*: In general, an I-channel is assigned to a client for VCR-like interactions. The CPE buffer can support restricted interactivity. Once the storage requirement of the client interaction exceeds the capacity of his CPE buffer, TVoD protocols should 'dispatch' an I-channel to support the client's interactions.

*Merging*: When the capacity of CPE buffer is exceeded, upon completing an interaction, the client needs to join a regular multicast channel, but s/he often cannot join an existing channel immediately because there may not be any channel with the desired playback time. In such a case, the client will be merged into the regular stream by using the assigned I-channel if available, or the protocol dispatches a new channel to the client for merging so that s/he can retain continuous interaction service.

In this paper, we focus on the performance analysis for the interaction and merging phases.

## III. PERFORMANCE ANALYSIS

### A. Customer Interaction Model

We use the interaction model proposed in [1] to analyze TVoD service. In this model, a set of states corresponding to different VCR actions are designed durations and probabilities of transitions to neighboring states. If the initial state is Play, then the system randomly transits to other interactive states or remains at Play state according to the behavior distribution. As shown in Figure 2, transition probabilities $P_i$ $(i = 0, \dots, 9)$ are assigned to a set of states corresponding to different VCR actions.

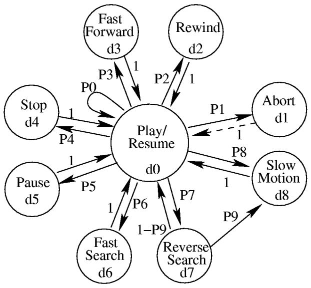

Figure 2. VCR interactive model

Assume that the system serves each state for an exponentially distributed period of time, and $d_i$ $(i = 0, 1, 2, \dots, 8)$ are the mean durations for the corresponding interaction states ($d_1 = 0$). Meanwhile, the speedup factors of Fast Forward/Rewind and Fast Search/Reverse Search are defined as $K_0$, $K_1$, respectively, and the speeddown factor of Slow Motion is defined as $K_2$.

### B. Analysis of Interaction Service

In order to analyze the interactivity, we introduce the following parameters summarized in Table 1.

*Interaction duration (ID), t*: the duration a user executes an interaction operation.

*Interaction effect offset (IEO), T*: the video offset an interaction makes. It reflects the real effect of an interaction.

*Interaction group offset (IGO), G*: the multicast group offset an interaction makes. In general, $G = T - t$.

Table 1. Interaction Parameters

| Action | ID($t$) | IEO ($T$) | IGO ($G$) |
|---|---|---|---|
| PLY | $t_0$ | $T_0 = t_0$ | $G_0 = 0$ |
| ABT | $t_1 = 0$ | $T_1 = 0$ | $G_1 = 0$ |
| REW | $t_2$ | $T_2 = -K_0 t_2$ | $G_2 = -(K_0 + 1)t_2$ |
| FF | $t_3$ | $T_3 = K_0 t_3$ | $G_3 = (K_0 - 1)t_3$ |
| STP | $t_4$ | $T_4 = 0$ | $G_4 = -t_4$ |
| PAU | $t_5$ | $T_5 = 0$ | $G_5 = -t_5$ |
| FS | $t_6$ | $T_6 = K_1 t_6$ | $G_6 = (K_1 - 1)t_6$ |
| RS | $t_7$ | $T_7 = -K_1 t_7$ | $G_7 = -(K_1 + 1)t_7$ |
| SM | $t_8$ | $T_8 = t_8 / K_2$ | $G_8 = (\frac{1}{K2} - 1)t_8$ |

#### B.1 Mean Number of Interactions

Let $N_0$ and $N_1$ denote the mean number of interactions for play without and with Abort, respectively, and $L$ is the length of the video. First, we try to find $N_0$. If the mean number of times entering each state with transition probability $P_i$ is $x_i$ $(i = 0, \dots, 7)$, we have $x_1 = 0$, and $x_8$ is the mean number of times SM state is entered either by the transition with probability $P_8$, or by the transition with probability $P_9$. According to the interaction model, we have

$$\begin{cases} E[T_0]x_0 + \sum_{i=2}^{8}(E[T_0] + E[T_i])x_i - P_9 E[T_0]x_7 = L & (1) \\ x_0 : x_i = P_0 : P_i, (i = 2, \dots, 7) & (2) \\ x_0 : x_8 = P_0 : (P_8 + P_9 P_7) & (3) \end{cases}$$

Thus, we can get the solutions of $x_i$ $(i = 0, \dots, 8)$: $x_0 = P_0 L / ((P_0 + P_8)E[T_0] + \sum_{i=2}^{7} P_i(E[T_0] + E[T_i]) + (P_8 + P_9 P_7)E[T_8])$; $x_i = P_i x_0 / P_0, (i = 2, \dots, 7)$; $x_8 = (P_8 + P_9 P_7)x_0 / P_0$. Thus,

$$N_0 = \sum_{i=2}^{8} x_i \tag{4}$$

Now, we consider the case with Abort. The probability of Abort occurrence once users interact with the system is

$$q = P_1 (\sum_{i=1}^{8} P_i)^{-1} \tag{5}$$

Because the occurrence of Abort action obeys a geometrical distribution with the density function $p(i) = q(1-q)^{i-1}, i = 1, 2, \dots$, among $N_0$ interactions, the probability of an Abort action occurring is approximated as

$$Q = (1 + \lfloor N_0 \rfloor - N_0) \sum_{i=1}^{\lfloor N_0 \rfloor} q(1-q)^{i-1} + (N_0 - \lfloor N_0 \rfloor) \sum_{i=1}^{\lfloor N_0 \rfloor + 1} q(1-q)^{i-1} \tag{6}$$

Meanwhile, $N_1$ equals the expectation

$$N_1 = [(1 + \lfloor N_0 \rfloor - N_0) \sum_{i=1}^{\lfloor N_0 \rfloor} iq(1-q)^{i-1} + (N_0 - \lfloor N_0 \rfloor) \sum_{i=1}^{\lfloor N_0 \rfloor + 1} iq(1-q)^{i-1}] / Q \tag{7}$$

The mean number of interactions ($N$) for each client, except Abort action, can be obtained by:

$$N = Q(N_1 - 1) + (1 - Q)N_0 \tag{8}$$

#### B.2 Mean actual playing time

The mean actual playing time ($l$) of each video is also divided into two cases: the playing time without Abort action (denoted as $l_{x_1=0}$) and the playing time when Abort action terminates playing (denoted as $l_{x_1=1}$).

First, we know

$$l_{x_1=0} = 2d_0 x_0 + \sum_{i=2}^{8}(d_0 + d_i)x_i \tag{9}$$

where $x_i$ $(i = 0, 2, \dots, 8)$ are found from Eqs. (1), (2) and (3).

Second, we consider the case when Abort action ends playing. Because the mean number of interactions is $N_1$, and $x_1 = 1$, we have $\sum_{i=2}^{8} x_i = N_1 - 1$. Combining this with Eqs. (2) and (3), we can get the solutions of $x_i$ in this case:

$x_0 = P_0(N_1 - 1)/(\sum_{i=2}^{8} P_i + P_9 P_7), \quad x_i = P_i x_0/P_0 (i = 2, \ldots, 8), \quad x_8 = (P_8 + P_9 P_7) x_0/P_0$.

Therefore,

$$l_{x_1=1} = \sum_{i=0}^{8} (d_0 + d_i) x_i \tag{10}$$

The mean actual playing time for each client is

$$l = Q l_{x_1=1} + (1 - Q) l_{x_1=0} \tag{11}$$

### B.3 Evaluating the I-channel requirement

Assume that the play point is located at the middle of the CPE buffer when the interaction begins, we have the probability that an interaction exceeds the capacity of the CPE buffer:

$$\begin{aligned}
g_2 &= P\{G_2 \le -B/2\} = exp(-\tfrac{B}{2(K_0+1)d_2}),\\
g_3 &= P\{G_3 \ge B/2\} = exp(-\tfrac{B}{2(K_0-1)d_3}),\\
g_4 &= P\{G_4 \le -B/2\} = exp(-\tfrac{B}{2d_4}),\\
g_5 &= P\{G_5 \le -B/2\} = exp(-\tfrac{B}{2d_5}),\\
g_6 &= P\{G_6 \ge B/2\} = exp(-\tfrac{B}{2(K_1-1)d_6}),\\
g_7 &= P\{G_7 \le -B/2\} = exp(-\tfrac{B}{2(K_1+1)d_7}),\\
g_8 &= P\{G_8 \le -B/2\} = exp(-\tfrac{K_2 B}{2(K_2-1)d_8}).
\end{aligned}$$

If $t_7$ and $t_8$ represent the interaction durations of RS, SM, respectively, they are exponentially-distributed random variables. We introduce two additional random variables $\xi_1$ and $\xi_2$:

$$\xi_1 = (K_1 + 1) t_7, \quad \xi_2 = \tfrac{(K_2-1)}{K_2} t_8.$$

Clearly, $\xi_1$ and $\xi_2$ are exponentially distributed with the density functions

$$f_1(z_1) = D_7 e^{-D_7 z_1}, \quad D_7 = \tfrac{1}{(K_1+1)d_7}$$

$$f_2(z_2) = D_8 e^{-D_8 z_2}, \quad D_8 = \tfrac{K_2}{(K_2-1)d_8}$$

Thus, we can find

$$\begin{aligned}
g_8' &= P\{G_7 + G_8 \le -B/2\} = 1 - P\{\xi < B/2\}\\
&= (D_8 e^{-D_7 B/2} - D_7 e^{-D_8 B/2})/(D_8 - D_7)
\end{aligned} \tag{12}$$

Moreover, if the IGO exceeds the size of the CPE buffer, we need to find the mean interaction duration of various actions so as to evaluate the channel requirement.

In order to evaluate the channel requirement when RS is transited to SM, we need to find the expectation $m$ of $\eta = t_7 + t_8$ if $G_7 + G_8 = -B/2$ $(t_7 \ge 0, t_8 \ge 0)$.

$$\begin{aligned}
m &= E[\eta | G_7 + G_8 = -B/2] = \tfrac{1}{K_1+1} E[\xi_1 | \xi_1 + \xi_2 = B/2]\\
&\quad + \tfrac{K_2}{K_2-1} E[\xi_2 | \xi_1 + \xi_2 = B/2]\\
&= \tfrac{1}{K_1+1} [\tfrac{B exp((D_8-D_7)B/2)}{2 exp((D_8-D_7)B/2)-2} - \tfrac{1}{D_8-D_7}]\\
&\quad + \tfrac{K_2}{K_2-1} [\tfrac{B exp((D_7-D_8)B/2)}{2 exp((D_7-D_8)B/2)-2} - \tfrac{1}{D_7-D_8}]
\end{aligned} \tag{13}$$

Further, the density function of $\eta$ is expressed as:

$$f(z) = \tfrac{1}{d_7-d_8} (e^{-\frac{z}{d_7}} - e^{-\frac{z}{d_8}}) \tag{14}$$

We want to obtain the mean interaction duration $\bar{t}_8'$ of transiting from RS to SM when IGO is greater than B/2, and it is easy to compute, but it is too tedious to express $\bar{t}_8' = E[\eta | G_7 + G_8 \le -B/2]$ clearly[1], so we use the following approximation:

$$\begin{aligned}
\bar{t}_8' &\approx E[\eta | \eta \ge m] = \int_m^{\infty} \frac{z}{d_7 - d_8} (e^{-\frac{z}{d_7}} - e^{-\frac{z}{d_8}}) dz / P(\eta \ge m)\\
&= \frac{d_7 (m + d_7) e^{-\frac{m}{d_7}} - d_8 (m + d_8) e^{-\frac{m}{d_8}}}{d_7 e^{-\frac{m}{d_7}} - d_8 e^{-\frac{m}{d_8}}}
\end{aligned} \tag{15}$$

We have the mean I-channel duration for an interaction

$$\begin{aligned}
c = [&g_6 P_6 d_6 + g_7 (1 - P_9) P_7 d_7 + g_8 P_8 d_8 +\\
&g_8' P_9 P_7 (\bar{t}_8' - m)] / \Sigma_{i=2}^{8} P_i
\end{aligned} \tag{16}$$

If the request rate is $\lambda$, the channel requirement of TVoD interaction service for one video is

$$C_{interaction} = N \lambda c / l \tag{17}$$

## C. *The merging channel requirement*

After completing an interaction, the client needs to continue using I-channel to join the nearest multicast group (see Figure 1). Assuming that $w$ is the grouping interval, the channel usage duration for merging is $a = w - mod(G + U, w)$, where $U \sim Uniform[0, w]$ is the group offset of the client before the interaction begins. So, the mean channel usage duration for one merging is $E[a] = w/2$.

If we use the SAM protocol, the merging channel requirement for one video

$$C_{merge} = N \lambda w / (2l) \tag{18}$$

We can find the probability of occurrence of merging the interaction stream with multicast streams

$$P_{merge} = (\sum_{i=2}^{8} g_i P_i + (g_8' - g_7) P_9 P_7) / \sum_{i=2}^{8} P_i \tag{19}$$

If we use the improved SAM protocol with the CPE buffer, the merging channel requirement for one video is

$$C_{merge} = P_{merge} N \lambda w / (2l) \tag{20}$$

## IV. EVALUATION WITH SIMULATIONS

We compare the channel requirements for the SAM protocol [6] and the SAM protocol improved by using the CPE buffer (abbreviated as BSM) [1]. We focus only on the comparison of their TVoD interactivities.

We studied the mean bandwidth requirement of a 90-minute video in our simulation. Requests arrive according to a Poisson process with rate ranging from 1 to 10 per minute. The patching window size is varied from 0 to 10 minutes, and the CPE buffer size is ranged from 0 to 10 minutes. Two types of interactive behaviors, VI and NVI, are simulated. The results are collected from 10-hour simulations. The speedup factors K0=10, K1=3, speeddown factor K2=2. The default values of the mean duration for various interaction states are given in Table 2. $d_i$ $(i = 1, \ldots, 8)$ in Table 3 are the default mean durations for various interactions, and the client with the duration factor $DF$ has the mean durations $DF * d_i$ $(i = 1, \ldots, 8)$.

Table 2. Transition probabilities

| Parameter | $P_0$ | $P_1$ | $P_2, P_3$ | $P_4$ | $P_5$ | $P_6, P_7$ |
|---|---|---|---|---|---|---|
| VI | 0.50 | 0.04 | 0.08 | 0.06 | 0.08 | 0.08 |
| NVI | 0.75 | 0.02 | 0.04 | 0.03 | 0.04 | 0.04 |

Table 3. Mean interactive durations

| Parameter | $d_0$ | $d_1$ | $d_2, d_3$ | $d_4, d_5$ | $d_6, d_7$ | $d_8$ |
|---|---|---|---|---|---|---|
| Default | 10 | 0 | 0.5 | 5 | 2.5 | 2 |

## A. *Mean Number of Interactions*

First, we check if the formula to compute the mean number of interactions matches the simulation results. The results should be related to the interaction frequencies (transition possibilities) and the mean durations of interactions. We compare both the computed and simulated results shown in Figure 3, and find them to be very close. We also notice that the mean number

---

[1] We can get the exact result of $\bar{t}_8'$ by Mathematica

of interactions does not change significantly when the mean durations vary because the forward and the backward actions have opposite effects on the playing times of videos.

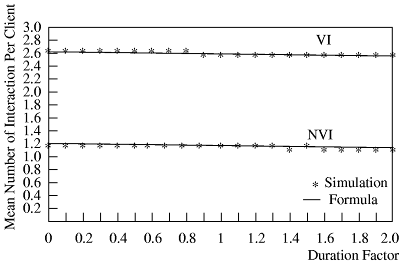

Figure 3. The mean number of interactions ($\lambda$=5)

### B. The I-Channel Requirements

The SAM protocol serves each interaction request by allocating an I-channel. Under the improved SAM, some of the interactions don't require any additional channel if they can be supported by the CPE buffer. Figure 4 shows the relationship between the request rate and the interaction channel requirement. The simulation results closely match the analysis results. Figure 5 indicates that the I-channel requirements are generally irrelevant to the grouping interval when the CPE buffer size and the request rate are fixed.

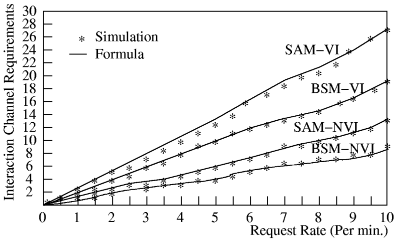

Figure 4. The request rate and I-channel requirements ($w$=5 min., $B$=5 min.)

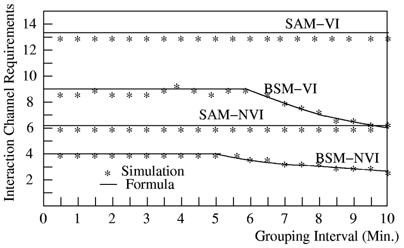

Figure 5. The grouping interval and the I-channel requirements ($\lambda$=5,If $w \leq 5$ min. $B = 5$ min.; if $w > 5$ min. $B = w$)

The simulation results also show that our formulae accurately estimate the relationship between the grouping interval and the I-channel requirements.

### C. The Merging Channel Requirements

We also evaluated the requirement of merging channels for VCR interactions in the VI/NVI mode. Figure 6 shows the merging channel requirement while varying the request rate. Figure 7 indicates that the grouping interval will greatly affect the merging channel requirement. Our analysis results match well the simulation results.

## V. CONCLUSIONS

Multicast is an excellent means to improve the efficiency of VoD systems. In this paper we quantified the resource requirement of multicast TVoD interactivity. We analyzed the user activity of multicast TVoD service, and built mathematical models to represent the channel requirement for multicast TVoD service protocols. The analytical results specify the relationships among the clients' behavior, the system resources in server and customer promise equipment, and the TVoD protocol. Our extensive simulations confirmed the correctness of our analysis results.

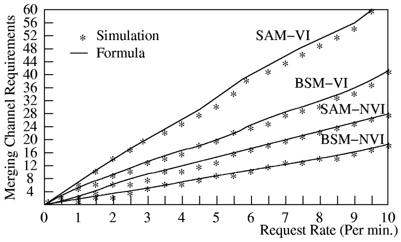

Figure 6. The request rate $\lambda$ and the merging channel requirements ($DF = 1, w$=5 min.)

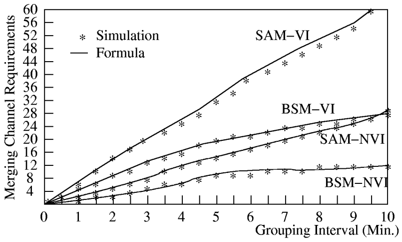

Figure 7. The grouping interval and the merging channel requirements ($\lambda = 5,\ DF = 1$. If $w \leq 5$ min. $B = 5$ min.; if $w > 5$ min. $B = w$)

## REFERENCES

[1] E. L. Abram-Profeta and Kang G. Shin, "Providing unrestricted VCR capability in multicast video-on-demand systems", *Proc. IEEE ICMCS'98*, TX, June-July 1998.

[2] K. C. Almeroth and M. H. Ammar, "The use of multicast delivery to provide a scalable and interactive video-on-demand service," *IEEE JSAC*, Vol.14, No.6, pp. 1110–1122, Aug.1996.

[3] R.O. Banker *et al.*, "Method of providing video-on-demand with VCR-like functions," *U.S. Patent 5357276*, 1994.

[4] A. Dan, *et al.*, "Channel allocation under batching and VCR control in video-on-demand systems," *Journal of Parallel and Distributed Computing*, 30:168-179, 1995.

[5] K. A. Hua, Y. Cai, and S. Sheu, "Patching: A multicast technique for true video-on-demand services," *Proc. ACM Multimedia'98*, pp. 191–200, Bristol, U.K., Sep. 1998.

[6] W. Liao and V.O.K. Li, "The split and merge protocol for interactive video-on-demand," *IEEE Multimedia*, Oct.-Dec. 1997, Pages 51-62.