# Constraint Design and Modelling of Multi-Hop Multi-Band Wireless MIMO Networks

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***Abstract*— Recent technological advances have enabled SDRs to switch from one frequency band to another at minimum cost, thereby making dynamic multi-band access and sharing possible. On the other hand, recent advances in signal processing combined with those in antenna technology provide MIMO capabilities, thereby creating opportunities for enhancing the throughput of wireless networks. Both SDRs and MIMO together enable next-generation wireless networks, such as mesh networks, to support dynamic and adaptive bandwidth sharing along time, frequency, and space. In this paper, we design and model the radio and interference constraints on multi-hop wireless networks when they are both SDR-capable and MIMO-equipped. The developed constraint models can be used for a variety of networking applications, such as network performance evaluation, routing algorithms, and admission control and QoS mechanisms.**

## I. Introduction

Recent advances in radio technologies have made it possible to realize SDRs (Software-Defined Radios) that, unlike traditional radios, can switch from one frequency band to another at no or little cost, thereby enabling dynamic and adaptive multi-band access and sharing. On the other hand, recent advances in signal processing combined with those in antenna technology empowered wireless networks with MIMO (Multiple-Input Multiple-Output) capabilities, thereby creating potential for network throughput enhancements via spatial reuse [1] and/or spatial multiplexing [2]. Therefore, SDR and MIMO are both considered as key wireless technological components that together form a complete means of enabling next-generation wireless networks with opportunistic bandwidth access and sharing along three dimensions: time, frequency, and space.

We design and model the radio and interference constraints on multi-hop wireless networks when they are both SDR-capable and MIMO-equipped. Deriving constraint models for these next-generation wireless networks can be used for a variety of objectives. For example, in order to evaluate the throughput performance of wireless networks, it often requires ways of modelling and capturing the radio and interference constraints [3]. Hence, the network constraint models (that we develop in this work) can serve as a means of identifying the limits and potential of SDRs and/or MIMO technologies in terms of the total throughput that they can provide to future networks, such as wireless mesh networks. Another networking problem where network constraint models can also be found useful is routing [4], [5]. For instance, linear programming-based routing formulations for multi-hop wireless networks require network constraint models that capture the radio and interference contention so that routing solutions are guaranteed to be feasible [4]. Modelling network constraints also presents a major challenge to QoS routing in multi-hop wireless networks [6]. Models that capture network constraints can be used to derive admission control mechanisms to support applications with QoS needs [7].

The rest of the paper is organized as follows. Section II describes the network model. Section III describes and models the radio and interference constraints on multi-hop, multi-band, wireless MIMO networks. In Section IV, we show how the derived constraint design can be used for throughput evaluation. We finally conclude in Section V.

## II. Model

### A. MIMO Model: Effective Degrees of Freedom (DoF)

MIMO offers great potentials for increasing the spatial reuse of the spectrum through better interference suppression capabilities [8]. When equipped with MIMO, transmitters can null their signals at undesired nearby receivers while ensuring acceptable signal gains at their desired receivers. Likewise, receivers can exploit their MIMO systems to suppress the interferences caused by the undesired nearby transmitters while successfully receiving their desired signals. Hence, MIMO can allow multiple simultaneous interference-free transmissions in the same vicinity, thereby potentially enhancing network throughput through spatial reuse.

The degree of realizing spatial reuse benefits offered by MIMO systems is contingent on physical limitations such as a node's transmission/reception power, multi-path, and channel coefficient estimation errors. For instance, suppose $m$ and $n$ are two neighbor nodes, equipped with an antenna array of size $\pi_m$ and $\pi_n$, respectively, and $m$ wants to

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transmit data to $n$. Assume that there are $\varphi$ communication streams currently being received by nodes located within $m$'s transmission range, and $\psi$ communication streams currently being transmitted by nodes located within $n$'s reception range. Due to physical limitations, the number $\varphi$ of nearby received streams that node $m$ can prevent its signal, being sent to $n$, from reaching at is (1) not proportional to, and (2) likely to be less than its actual number of antennas $\pi_m$ [8]. The number $\theta_m \equiv (\varphi + 1)$ is referred to as $m$'s *effective transmit DoF* (1 corresponds to the communication stream from $m$ to $n$). For similar reasons, the number $\vartheta_n \equiv (\psi + 1)$ of possible concurrent streams in $n$'s vicinity, referred to as $n$'s *effective receive DoF*, is (1) not proportional to, and (2) also likely to be less than $n$'s total number of antennas $\pi_n$ [8]. In [9], we derived a table-driven statistical method that allows each node $m$ to determine both $\theta_m$ and $\vartheta_m$ given the network's physical constraints. In this paper, we assume that nodes use this method to determine their effective transmit and receive DoFs.

### B. Network Model

We assume that the radio spectrum is divided into multiple non-overlapping bands, and $K$ is the set of these spectrum bands. A multi-hop wireless network is modeled as a directed graph $G = (N, L)$ with a finite nonempty set $N$ of nodes and a finite set $L$ of wireless data links. $L$ is the set of all ordered pairs $(m, n)$ of distinct nodes in $N$ such that $n$ is within $m$'s transmission range. If $i = (m, n) \in L$, then $m$ and $n$ are referred to as the transmitter $t(i)$ and the receiver $r(i)$ of link $i$, respectively. A data link $i$ is said to be *active* if $t(i)$ is currently transmitting to $r(i)$; otherwise, $i$ is said to be *inactive*. For every $m \in N$, let $L_m^+ = \{i \in L : t(i) = m\}$, $L_m^- = \{i \in L : r(i) = m\}$, and $L_m = L_m^+ \cup L_m^-$. We assume that each node $m$ is equipped with an antenna array of $\pi_m$ elements, and let $\theta_m$ and $\vartheta_m$ denote the effective transmit and receive DoFs of $m$. For every $(i, k) \in L \times K$, let $c_{ik}$—which is assumed to be time-invariant—denote the maximum number of bits that link $i$ can support in 1 second if communicated on spectrum band $k$.

Let $C$ denote the set of all distinct ordered pairs $(i, j) \in L \times L$ such that (1) $i$ and $j$ do not share any node between them and (2) the transmission on link $i$ interferes with the reception on link $j$ when communicated on the same spectrum band. Note that $(i, j) \in C$ does not necessarily imply that $(j, i) \in C$. For every link $i \in L$, let $C_i^+ = \{j \in L : (i, j) \in C\}$ denote the set of all links whose receivers interfere with the transmission on $i$, and $C_i^- = \{j \in L : (j, i) \in C\}$ denote the set of all links whose transmitters interfere with the reception on $i$.

We assume that a node can either transmit or receive, but not both, at any time. We also assume that each link can be active on at most one band at a time. A link can, however, be active on two different bands during two different time slots. We consider the TDMA scheme to share the wireless medium. Time is then divided into time slots of an equal length. Let $T = \{1, 2, \ldots\}$ denote the set of these time slots.

## III. Constraint Design

In this section, we describe and model the radio and interference constraints on the multi-hop, multi-band wireless MIMO networks, described in Section II. For every $(i, k, t) \in L \times K \times T$, we define the binary variable $y_{ik}^t$ to be 1 if link $i$ is active on spectrum band $k$ during time slot $t$, and 0 otherwise.

### A. Packet-Level Constraints

*1) Radio Constraints:* We assume that a link can be active on at most one spectrum band at any given time slot, i.e.,

$$\sum_{k \in K} y_{ik}^t \leq 1, \forall i \in L, \forall t \in T.$$

Due to radio constraints, we also assume that a node can either transmit or receive, but not both, at any time slot; that is, for all $i \in L$ and for all $k \in K$,

$$\sum_{j \in L_{t(i)}^-} y_{jk}^t \leq M(1 - y_{ik}^t) \text{ and } \sum_{j \in L_{r(i)}^+} y_{jk}^t \leq M(1 - y_{ik}^t)$$

where $M = |L|$ is an integer larger than the maximum number of active links at any time $t$. Recall that with MIMO systems, a node uses one DoF (degree of freedom) to transmit or receive a desired signal while using the other DoFs to allow for multiple simultaneous nearby communication sessions; that is, for all $m \in N$, for all $k \in K$, and for all $t \in T$,

$$\sum_{j \in L_m^-} y_{jk}^t \leq 1 \text{ and } \sum_{j \in L_m^+} y_{jk}^t \leq 1.$$

All of the above constraints can be equivalently written as

$$\sum_{k \in K} \sum_{i \in L_m} y_{ik}^t \leq 1, \quad \forall m \in N, \forall t \in T \tag{1}$$

*2) Interference Constraints:* We now describe and model the interference constraints. Recall that each receiver must have enough effective receive DoF that enable it to combat the interference caused by all nearby transmitters prior to receiving its desired signal at any time slot, i.e., $\forall i \in L, \forall k \in K, \forall t \in T$,

$$(M - \vartheta_{r(i)} + 1) y_{ik}^t + \sum_{j \in C_i^-} y_{jk}^t \leq M \tag{2}$$

where again $M = |L|$. If $y_{ik}^t = 1$ (i.e., $i$ is active), then the above constraints ensure that the total number of active links, interfering with the reception on link $i$ on spectrum band $k$, does not exceed what node $r(i)$'s effective receive DoF could handle; otherwise (if $y^t_{jk} = 0$), the constraints are relaxed since $i$ is not active, and hence, no interference needs to be suppressed.

Likewise, transmitters must also be responsible for nulling their signals at all nearby receivers. That is, prior to transmission at any slot time, a transmitter must have enough effective transmit DoF so that it can prevent its signal from causing interference to any nearby receivers. Hence, for all $i \in L$, for all $k \in K$, and for all $t \in T$,

$$(M - \theta_{t(i)} + 1)y^t_{ik} + \sum_{j \in C^+_i} y^t_{jk} \leq M. \tag{3}$$

Again, the above constraints ensure that the maximum number of active links that interfere with the transmission on link $i$ does not exceed what node $t(i)$ can null, i.e., no more than $\theta_{t(i)}$ can be concurrently active at time slot $t$ on the same spectrum band $k$ when $t(i)$ is active. However, if $t(i)$ is not transmitting, then the constraints should be relaxed as expressed by the inequality via $M$.

### B. Flow-Level Constraints

Note that the packet-level constraints, described in Section III-A, are (1) not linear (expressed in binary variables) and (2) instantaneous (expressed on a packet-by-packet basis). While the non-linearity feature prevents the use of standard LP methods to solve our multi-commodity routing problem, the packet-level granularity increases the size of the problem in terms of both number of equations and number of variables. These two features render the problem too complex to solve.

To reduce the complexity of the problem, we propose to LP-relax the packet-level constraints. As it will become clear shortly, the LP-relaxed constraints can be viewed as necessary conditions on feasibility of the *average* link rates. It is important to recall that LP relaxations result in widening the feasibility space, i.e., the solutions obtained under the average rate (relaxed) constraints may be infeasible under the instantaneous rate constraints. However, since we seek to characterize the maximum throughput, these relaxations will only make the maximum less tight. There is a clear tradeoff between the solution-quality and the problem-size/complexity. To keep the problem simple while drawing useful conclusions, we use the LP-relaxed constraints instead.

Let's consider a set of time slots $S \subseteq T$ of cardinality $\tau = |S|$, and define a continuous variable $\rho_{ik}$ to be

$$\rho_{ik} = \tfrac{1}{\tau}\sum_{t \in S} y^t_{ik}, \quad \forall i \in L, \forall k \in K.$$

Note that $\rho_{ik}$ represents the fraction of time in $S$ during which link $i$ is active on band $k$. Recall that this continuous variable is *averaged* over the length of the time slot set $S$. Hence, the longer $S$ is, the more accurate this average becomes. Throughout the rest of this paper, we assume that the length of $S$ is long enough for these variables to reflect accurate averages.

By using this continuous variable, one can provide LP relaxations to the packet-level constraints described in III-A. For example, by summing both sides of Eq. (1) over $S$ and interchanging summations between $k$ and $t$, one can obtain

$$\sum_{k \in K}\sum_{i \in L_m} \rho_{ik} \leq 1, \quad \forall m \in N. \tag{4}$$

When applying the same technique to the interference constraints, given by Eqs. (2) and (3), one can obtain the following LP-relaxed interference constraints.

$$\begin{cases} (M - \vartheta_{r(i)} + 1)\rho_{ik} + \sum_{j \in C^-_i} \rho_{jk} \leq M \\ (M - \theta_{t(i)} + 1)\rho_{ik} + \sum_{j \in C^+_i} \rho_{jk} \leq M \end{cases} \tag{5}$$

for all $(i, k) \in L \times K$.

## IV. Throughput

Let's now illustrate how one would use the derived network constraint design to determine routing solutions for next-generation wireless networks. Let's consider a multi-band, multi-hop wireless MIMO routing instance that consists of a set $Q$ of commodities, and let $x^q_{ik}$ denote link $i$'s data rate on band $k$ that belongs to commodity $q$. Note that the flow balance constraints

$$\sum_{j \in L^+_{t(i)}}\sum_{k \in K} x^q_{jk} = \begin{cases} f_q & \text{if } t(i) = s(q) \\ \sum_{k \in K}\sum_{j \in L^-_{t(i)}} x^q_{jk} & \text{otherwise,} \end{cases} \tag{6}$$

must be satisfied for all $i \in L$ and all $q \in Q$. Also, note that for all $(i, k) \in L \times K$,

$$\rho_{ik} = \frac{1}{c_{ik}}\sum_{q \in Q} x^q_{ik}. \tag{7}$$

We now formulate the multi-hop, multi-band, wireless MIMO network routing problem as a standard multi-commodity flow instance that consists of a set $Q$ of commodities where each $q \in Q$ is characterized with a source-destination pair $s(q), d(q) \in N$ and a non-negative multi-hop flow of rate $f_q$. A multi-hop flow solution—maximizing the sum $\sum_{q \in Q} f_q$ of all flows' rates subject to the constraints expressed via Eqs. (4), (5), (6), and (7)—can be used to signify the total achievable throughput under multi-commodity flow $f = (f_q)_{q \in Q}$. By solving many instances, one can then characterize the maximum throughput that multi-hop, multi-band, wireless MIMO networks can achieve. The use of this routing formulation to characterize the total achievable throughput in multi-hop, multi-band, wireless MIMO networks is currently underway.

## V. Conclusion

We describe and model the radio and interference constraints on multi-hop wireless networks when they are both multi-band-capable and MIMO-equipped. The developed framework can be used to identify the limits and potentials of key technological components, such as SDR and MIMO, in terms of the maximum achievable throughput of next-generation wireless networks when equipped with these new technologies. We illustrate how one can use the developed network constraints to formulate and solve multi-hop routing problems in wireless networks with MIMO and multi-band capabilities.

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