Maximum Achievable Throughput in Multiband Multiantenna Wireless Mesh Networks

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Abstract—We have recently witnessed a rapidly increasing demand for, and hence, a shortage of, wireless network bandwidth due to rapidly growing wireless services and applications. It is, therefore, important to develop an efficient way of utilizing this limited bandwidth resource. Fortunately, recent technological advances have enabled software-defined radios (SDRs) to switch from one frequency band to another at minimum cost, thereby making dynamic multiband access and sharing possible. On the other hand, recent advances in signal processing combined with those in antenna technology provide multiple-input multiple-output (MIMO) capabilities, thereby creating opportunities for enhancing the throughput of wireless networks. Both SDRs and MIMO together enable next-generation wireless networks, such as mesh networks, to support dynamic and adaptive bandwidth sharing along time, frequency, and space. In this paper, we develop a new framework that 1) identifies the limits and potential of SDRs and MIMO in terms of achievable network throughput and 2) provides guidelines for designers to determine the optimal parameters of wireless mesh networks equipped with multiband and multiantenna capabilities.

Index Terms—Maximum throughput, multiantenna systems, multiband access, network modeling and design, wireless mesh networks.

1 Introduction

The rapidly growing popularity of wireless technology has recently generated an explosive demand for wireless network bandwidth. The bandwidth supply, on the other hand, has not kept up with this fast-growing demand. This expected shortage of bandwidth has prompted both industry [1], [2], [3] and government [4], [5] to explore new ways of efficiently using this limited resource.

Fortunately, recent advances in radio technologies have made it possible to realize Software-Defined Radios (SDRs) that, unlike traditional radios, can switch from one frequency band to another at no or little cost, thereby enabling dynamic and adaptive multiband access and sharing. SDRs are considered as a key next-generation wireless technology to improve bandwidth utilization. On the other hand, recent advances in signal processing combined with those in antenna technology empowered wireless networks with Multiple-Input Multiple-Output (MIMO) or multiantenna capabilities, thereby creating potential for network throughput enhancements via spatial reuse [6] and/or spatial multiplexing [7]. Therefore, SDR and MIMO complement each other to form a complete means of enabling next-generation wireless networks with opportunistic bandwidth utilization along not only time and frequency dimensions via SDRs, but also space dimension via MIMO.

Wireless mesh networks (WMNs) have also been considered as a key wireless networking technology for their advantages over traditional wireless networks, such as low cost, easy installation and maintenance, robustness, and reliability [8], [9], [10]. In addition to these capabilities, WMNs can still exploit SDRs and MIMO to increase their total throughput, thereby improving spectrum efficiency even further.

In this paper, we develop a framework that 1) identifies the limits and potential of SDRs and MIMO technologies in terms of the total throughput that they can provide to WMNs and 2) derives guidelines for designing and optimizing multiband-capable, multiantenna-equipped WMNs. While SDRs are used to enable WMNs with dynamic and adaptive multiband access, MIMO systems are used to increase the spatial reuse of spectrum, and hence, the total network throughput. It is important to note that, although MIMO can be exploited to increase the overall network throughput via not only spatial reuse but also spatial multiplexing, we will focus on MIMO's spatial reuse capabilities, leaving the problem of exploiting MIMO to increase network throughput via spatial multiplexing as our future work.

The rest of the paper is organized as follows: Section 2 discusses the related work, putting our work in a comparative perspective. Section 3 illustrates how spatial reuse can be increased with MIMO. Section 4 describes the network model, states our objective, and outlines the proposed approach. Section 5 models the radio and interference constraints. In Section 6, we formulate the WMN routing problem and propose a fast solution algorithm. Section 7 identifies the maximum achievable throughput in WMNs and derives design guidelines. We finally conclude the paper in Section 9.

2 Related Work

The apparent promise of SDRs has prompted researchers to think of ways of using them to enhance spectrum efficiency. As a result, there have recently been numerous publications

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addressing SDR-related challenges [11], [12], [13], [14], [15], [16]. Most of these papers aim to improve the spectrum efficiency along time and frequency dimensions via 1) adaptive and dynamic multiband access, 2) spectrum sharing among different users, and 3) coordination among different users for better spectrum utilization. Several researchers have also attempted to characterize throughput/capacity of wireless networks when nodes are equipped with single antennas [17], [18], [13], [19], [20], [21]. Gupta and Kumar [17] derived the asymptotic capacity of multihop wireless networks of static nodes, each equipped with a single omnidirectional antenna. The work in [18] shows that per-user throughput can increase dramatically when nodes are mobile rather than fixed by exploiting a form of multiuser diversity via packet relaying. Several other studies have also focused on characterizing the capacity in multichannel wireless networks [13], [19], [20], [21]. The work in [17] has been extended in [13] to multichannel wireless networks, where nodes, each equipped with multiple interfaces, cannot have a dedicated interface per channel. Their results show that the capacity of such networks depends on the ratio of the number of channels to the number of interfaces. Alicherry et al. [19] developed a solution for routing in multichannel, multi-interface wireless mesh networks that maximizes the overall network throughput subject to fairness and interference constraints. The authors in [20], [21] derived necessary and sufficient conditions for the feasibility of rate vectors in multiband, multiradio/interface WMNs, and used them to find upper bounds on the achievable throughput. Unlike these previous studies, we consider the throughput of multihop, multichannel networks, also equipped with MIMO links.

For their potential benefits, MIMO or multiantenna systems have also attracted considerable attention, yielding numerous proposals of MIMO-based techniques for single-band wireless networks [6], [7], [22], [23]. Most of these consist of designing MAC protocols that exploit the benefits of MIMO to enhance the network capacity [6], increase the data rates [7], and/or reduce energy consumption [22]. In [23], we derived a framework that characterizes the total achievable throughput in multiantenna-equipped WMNs when they are allowed to communicate on single band only. However, little has been done on how to exploit a combination of SDRs and MIMO to enhance spectrum efficiency along all three dimensions of time, frequency, and space. We adapt the LP constraint relaxation technique from [20] to characterize and analyze the maximum achievable throughput that multihop, multiband wireless networks can achieve when they are also equipped with MIMO links.

3 Spatial Reuse

Consider the example WMN in Fig. 1 that consists of four nodes, A, B, C, and D. Assume that there are only two concurrent transmissions: $A \rightarrow B$ and $C \rightarrow D$. As depicted in the figure, A's transmitted signal is assumed to reach not only the desired receiver B but also the undesired receiver D. First, note that if the nodes are equipped with single omnidirectional antennas, then A's transmission will interfere with D's reception. Hence, D will not be able to successfully receive its intended signal from C. Here, we

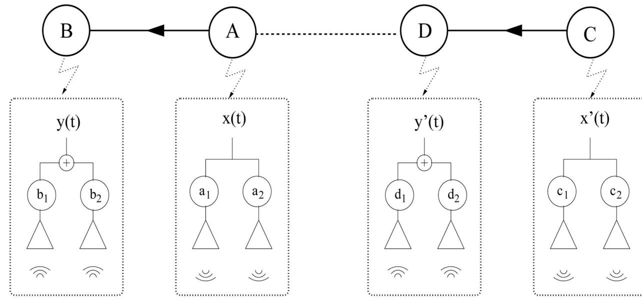

Fig. 1. Realizing spatial reuse via multiantenna systems.

illustrate how multiantenna systems can be exploited to allow for multiple simultaneous transmissions in the same neighborhood. That is, we will show that with two or more antennas, D can successfully receive its desired signal from C concurrently with A's undesired transmission. For illustration purpose, we assume that each node is equipped with two antennas.

In order to communicate with B, node A uses its two antennas to send two weighted copies of its signal $x(t)$. Let $a_1x(t)$ and $a_2x(t)$ denote the copies sent on antenna 1 and antenna 2, respectively; we refer to $\mathbf{a} = [a_1\ a_2]^T$ as node A's *transmission vector* (see Fig. 1). The receiver B constructs its desired signal by first weighing the two received signals with its reception vector $\mathbf{b} = [b_1\ b_2]^T$ and then summing them up to generate $y(t)$. Let $\mathbf{H}_{a,b}$ denote the matrix of channel coefficients between the transmitter A and the receiver B, then one can write $y(t) = (\mathbf{a}^T\mathbf{H}_{a,b}\mathbf{b})x(t)$. Now, let $\mathbf{c} = [c_1\ c_2]^T$ and $\mathbf{d} = [d_1\ d_2]^T$ denote, respectively, node C's transmission and node D's reception vectors. Because node D is within the transmission ranges of both A and C, its received signal $y'(t)$ can be expressed as $y'(t) = (\mathbf{c}^T\mathbf{H}_{c,d}\mathbf{d})x'(t) + (\mathbf{a}^T\mathbf{H}_{a,d}\mathbf{d})x(t)$, where $\mathbf{H}_{c,d}$ and $\mathbf{H}_{a,d}$ are the channel coefficient matrices between node D and its immediate neighbors C and A, respectively. Knowing $\mathbf{H}_{a,d}$, $\mathbf{a}$, $\mathbf{H}_{c,d}$, and $\mathbf{c}$, node D can choose its reception vector $\mathbf{d}$ so that it may receive 1) a unit gain signal from its intended transmitter C by ensuring that $(\mathbf{c}^T\mathbf{H}_{c,d}\mathbf{d}) = 1$, and 2) a zero gain signal from the undesired transmitter A by ensuring that $(\mathbf{a}^T\mathbf{H}_{a,d}\mathbf{d}) = 0$. Hence, with multiantenna systems, a node can receive an interference-free signal from its desired transmitter *concurrently* with nearby undesired transmitted signals. It is important to note that for the sake of keeping the illustration simple and focused, the analysis provided in this section intentionally assumes that 1) the matrices of channel coefficients are all of full rank and 2) there is no power limitation. In fact, if one or both of these two assumptions is relaxed, D may still not be able to receive an interference-free, desired signal even if it is equipped with two antennas. The effect of physical limitations, such as power and channel coefficients, is addressed in Section 4.1.

In summary, multiantenna systems can be exploited by transmitters to null their signals at undesired nearby receivers while ensuring acceptable signal gains at their desired receivers. Likewise, receivers can exploit their multiantenna systems to suppress the interferences caused by the undesired nearby transmitters while successfully receiving their desired signals. Multiantenna systems can thus allow multiple simultaneous interference-free transmissions in the same vicinity, thereby potentially enhancing network throughput. This is known as *spatial reuse*.

4 Problem Statement

We now describe the system model, state our objective, and outline the proposed approach to achieve the objective.

4.1 Effective Degrees of Freedom (DoF)

The degree of realizing spatial reuse benefits offered by multiantenna systems is contingent on physical limitations such as a node's transmission/reception power, multipath, and channel coefficient estimation errors. For instance, suppose that m and n are two neighbor nodes, equipped with an antenna array of size π_m and π_n, respectively, and m wants to transmit data to n. Assume that there are φ communication streams currently being received by nodes located within m's transmission range, and ψ communication streams currently being transmitted by nodes located within n's reception range. Due to physical limitations, the number φ of nearby received streams that node m can prevent its signal, being sent to n, from reaching is 1) not proportional to and 2) likely to be less than its actual number of antennas π_m [24]. The number $\theta_m \equiv (\varphi + 1)$ is referred to as m's *effective transmit DoF* (1 corresponds to the communication stream from m to n). For similar reasons, the number $\vartheta_n \equiv (\psi + 1)$ of possible concurrent streams in n's vicinity, referred to as n's *effective receive DoF*, is 1) not proportional to and 2) also likely to be less than n's total number of antennas π_n [24].

In [25], we derived a table-driven statistical method that allows each transmitter m and each receiver n to determine θ_m and ϑ_m, given the network's physical constraints. We assume that nodes use this method to determine their effective transmit and receive DoFs. For completeness, we briefly describe this method (its details can be found in [25]). As shown in [25], θ_m depends on 1) the transmitter's level of available power P_m, 2) the error variance associated with the channel estimation method σ_E^2, and 3) the receiver n's number of neighbors $\kappa(n)$. The method consists of dividing P_m into three levels: LOW, MEDIUM, and HIGH; σ_E^2 into three categories: ERRONEOUS, GOOD, and PERFECT; and $\kappa(n)$ into three types: DENSE, AVERAGE, and SPARSE. Each transmitter maintains a three-dimensional table, whose entries can be computed offline using equations derived in [25], which can be indexed by the three parameters, P_m, σ_E^2, and $\kappa(n)$, to determine θ_m. The idea here is that, by monitoring P_m, σ_E^2, and $\kappa(n)$, m can use its table to determine its effective transmit DoFs in real time. A receiver applies a similar method to determine its effective receive DoFs.

Note that we use these effective transmit and receive DoFs as a means of modeling the cross-layer effects of the nodes' and network's physical limitations on the transmission and reception capabilities of multiantenna systems—they capture the effects of the nodes' power availability, the multipath nature of a wireless environment, and the coefficients of a wireless channel [24].

Fig. 2. Illustrative topology.

4.2 Cooperative versus Noncooperative

A transmitter m's effective transmit DoFs can be viewed as m's number of transmitted streams plus the maximum number of streams that m can prevent its signal from reaching, i.e., those streams that are received within m's transmission range, and hence, interfere with m's transmitted signal. Similarly, a receiver n's effective receive DoFs can be viewed as n's number of received streams plus the maximum number of streams (those transmitted within n's reception range) that n can suppress.

There are two approaches that nodes can use to suppress/null interference through the exploitation of their effective DoFs: noncooperative and cooperative. The former requires that 1) new transmitters be responsible for nulling their signals at all nearby interfering receivers prior to transmitting their signals and 2) new receivers be responsible for suppressing the interference caused by all nearby transmitters prior to receiving their desired signals. That is, before transmitting its signal, a transmitter must ensure that it has enough effective transmit DoFs to transmit the signal without causing interference to any of its nearby receivers. Likewise, prior to receiving signals, a receiver must ensure that it has enough effective receive DoFs to be able to suppress the interference caused by all nearby transmitters while receiving its desired signals without interference. Referring to the topology given in Fig. 2 as an example (node 2 sends to node 4, and node 1 sends to node 3), under the noncooperative approach, node 4 must then be able to suppress node 1's signal prior to receiving node 2's signal, and node 1 must be able to null its signal at node 4 prior to transmitting a signal to node 3.

The cooperative approach, on the other hand, requires that either the transmitter or the receiver (but not necessarily both) be responsible for interference avoidance. For example, when referring to the same example in Fig. 2, the cooperative approach requires that either node 4 suppresses node 1's signal, or node 1 nulls its signal at node 4. Note that it suffices for node 4 to suppress node 1's signal, or for node 1 to null its signal at node 4 to have two successful transmissions. Thus, one DoF can be saved/used for suppressing/nulling other signals. Clearly, the cooperative approach allows for more concurrent communication streams. It is, however, more complex and incurs more overhead due to cooperation. The noncooperative approach, on the other hand, is more conservative, but less complex. In this paper, we assume the noncooperative approach.

4.3 Network Model

We assume that the radio spectrum is divided into multiple nonoverlapping bands, and K is the set of these spectrum bands. A WMN is modeled as a directed graph $G = (N, L)$ with a finite nonempty set N of nodes and a finite set L of wireless data links. L is the set of all ordered pairs (m, n) of distinct nodes in N such that n is within m's transmission range. If $i = (m, n) \in L$, then m and n are referred to as the transmitter $t(i)$ and receiver $r(i)$ of link i, respectively. A data link i is said to be *active* if $t(i)$ is currently transmitting to $r(i)$; otherwise, i is said to be *inactive*. For every $m \in N$, let $L_m^+ = \{i \in L : t(i) = m\}$, $L_m^- = \{i \in L : r(i) = m\}$, and $L_m = L_m^+ \cup L_m^-$. We assume that each node m is equipped with an antenna array of π_m elements, and let θ_m and ϑ_m denote the effective transmit and receive DoFs of m, respectively. For every $(i,k) \in L \times K$, let c_{ik}—which is assumed to be time invariant—denote the maximum number of bits that link i can support in 1 second if communicated on spectrum band k.

Let C denote the set of all distinct ordered pairs $(i,j) \in L \times L$ such that 1) i and j do not share any node between them and 2) the transmission on link i interferes with the reception on link j when communicated on the same spectrum band. Note that $(i,j) \in C$ does not necessarily imply that $(j,i) \in C$. For every link $i \in L$, let $C_i^+ = \{j \in L : (i,j) \in C\}$ denote the set of all links whose receivers interfere with the transmission on i, and $C_i^- = \{j \in L : (j,i) \in C\}$ denote the set of all links whose transmitters interfere with the reception on i.

We assume that a node can either transmit or receive, but not both, at any time. We also assume that each link can be active on at most one band at a time. A link can, however, be active on two different bands during two different time slots. We consider the TDMA scheme to share the wireless medium. Time is then divided into time slots of an equal length. Let $T = \{1, 2, \ldots\}$ denote the set of these time slots. The throughput achievable under TDMA will then be viewed as an upper bound on those achievable under other multiple access methods such as CDMA and CSMA/CA. It is important to reiterate that our goal is to characterize the maximum achievable network throughput. That is, how to achieve this maximum throughput is of no relevance to our work, and so are the details regarding the TDMA scheme, such as time synchronization and overhead.

4.4 Objective and Approach

First, we characterize and analyze the throughput that WMNs can achieve when they are 1) equipped with multiple antennas and 2) capable of communicating on multiple spectrum bands. We begin with the development of a model that captures the radio and interference constraints on multiband-capable, multiantenna-equipped WMNs. We then formulate the WMN routing problem as a standard multicommodity instance, consisting of a set Q of end-to-end flows, where each flow $q \in Q$ is characterized with a source-destination pair $s(q), d(q) \in N$, and a non-negative rate f_q. The WMN routing problem is then written as a packing LP whose objective is to maximize the sum of all flows, $\sum_{q \in Q} f_q$, subject to network constraints that we describe and model in Section 5. The sum $\sum_{q \in Q} f_q$ will be used to signify the maximum achievable throughput under a multicommodity flow f. We also propose a fast algorithm that finds a $(1-\epsilon)^{-2}$-approximation to the multicommodity flow optimal solution (in minimizing the running time) that depends polynomially on ϵ^{-1}. The input parameter ϵ can be appropriately fixed so that a solution with acceptable quality can be obtained in polynomial time. By solving many instances, we can then identify the maximum throughput these WMNs can achieve.

Second, based on the thus-obtained results and analysis of the achievable throughput, we derive guidelines for designing multiband, multiantenna WMNs. We first study the effects of transmission ranges and node degrees on the maximum achievable throughput. We then demonstrate how the designers can use the end results of this study to determine the optimal network parameters, such as transmission powers and node densities, that maximize the overall achievable throughput of a WMN.

5 Network Constraints

In this section, we describe and model the radio and interference constraints on the multiband, multiantenna WMN, described in Section 4. For every $(i,k,t) \in L \times K \times T$, let us define the binary variable y_{ik}^t to be 1 if link i is active on spectrum band k during time slot t, and 0 otherwise.

5.1 Packet-Level Constraints

5.1.1 Radio Constraints

We assume that a link can be active on at most one spectrum band at any given time slot, i.e., $\sum_{k \in K} y_{ik}^t \leq 1, \forall i \in L, \forall t \in T$. Due to radio constraints, we also assume that a node can either transmit or receive, but not both, at any time slot, i.e., $\forall i \in L, \forall k \in K$,

$$\sum_{j \in L_{t(i)}^-} y_{jk}^t \leq M(1 - y_{ik}^t) \text{ and } \sum_{j \in L_{r(i)}^+} y_{jk}^t \leq M(1 - y_{ik}^t),$$

where $M = |L|$ is an integer larger than the maximum number of active links at any time t. Let us consider the first set of constraints (left inequalities) for illustration. For a given flow i, this set ensures that if the transmitter $t(i)$ of flow i is transmitting (i.e., flow i is active) at time t on band k, then $t(i)$ cannot be the receiver of any flow j. In equation terms, if $y_{ik}^t = 1$ (i.e., flow i is active at time t on band k), then $\sum_{j \in L_{t(i)}^-} y_{jk}^t \leq 0$ (i.e., none of the flows j whose receiver is $t(i)$ can be active, meaning that $t(i)$ cannot be receiving while transmitting). Now, if flow i is not active (i.e., $y_{ik}^t = 0$), then the constraints must be relaxed, i.e., there should be no constraints. Indeed, when $y_{ik}^t = 0$, the constraints become $\sum_{j \in L_{t(i)}^-} y_{jk}^t \leq M$, and by setting $M = |L|$, such inequalities become constraint-free. Likewise, the right set of inequalities ensures that when a node is receiving at a given time slot, it cannot be transmitting during that same time slot.

Recall that with multiantenna systems, a node uses one degree of freedom (DoF) to transmit or receive a desired signal while using the other DoFs to allow for multiple simultaneous nearby communication sessions, i.e., $\forall m \in N, \forall k \in K, \forall t \in T$, $\sum_{j \in L_m^-} y_{jk}^t \leq 1$, and $\sum_{j \in L_m^+} y_{jk}^t \leq 1$. All of the above constraints can be equivalently written as

$$\sum_{k \in K} \sum_{i \in L_m} y_{ik}^t \leq 1, \quad \forall m \in N, \forall t \in T. \tag{1}$$

5.1.2 Interference Constraints

We now describe and model the interference constraints. Recall that each receiver must have enough effective receive DoFs that enable it to combat the interference caused by all nearby transmitters prior to receiving its desired signal at any time slot, i.e., $\forall i \in L, \forall k \in K, \forall t \in T$,

$$(M - \vartheta_{r(i)} + 1) y_{ik}^t + \sum_{j \in C_i^-} y_{jk}^t \leq M, \tag{2}$$

where again $M = |L|$. If $y_{ik}^t = 1$ (i.e., i is active), then the above constraints ensure that the total number of active links, interfering with the reception on link i on spectrum band k, does not exceed what node $r(i)$'s effective receive DoF could handle; otherwise (if $y_{jk}^t = 0$), the constraints are relaxed since i is not active, and hence, no interference needs to be suppressed.

Likewise, transmitters must also be responsible for nulling their signals at all nearby receivers. That is, prior to transmission at any slot time, a transmitter must have enough effective transmit DoFs so that it can prevent its signal from causing interference to any nearby receivers. Hence, $\forall i \in L, \forall k \in K, \forall t \in T$,

$$(M - \theta_{t(i)} + 1)y_{ik}^t + \sum_{j \in C_i^+} y_{jk}^t \leq M. \tag{3}$$

Again, the above constraints ensure that the maximum number of active links that interfere with the transmission on link i does not exceed what node $t(i)$ can null, i.e., no more than $\theta_{t(i)}$ can be concurrently active at time slot t on the same spectrum band k when $t(i)$ is active. However, if $t(i)$ is not transmitting, then the constraints should be relaxed as expressed by the inequality via M.

5.2 Flow-Level Constraints

Note that the packet-level constraints, described in Section 5.1, are 1) not linear (expressed in binary variables) and 2) instantaneous (expressed on a packet by packet basis). While the nonlinearity feature prevents the use of standard LP methods to solve our multicommodity routing problem, the packet-level granularity increases the size of the problem in terms of both number of equations and number of variables. These two features render the problem too complex to solve.

To reduce the complexity of the problem, we propose to LP-relax the packet-level constraints. As it will become clear shortly, the LP-relaxed constraints can be viewed as necessary conditions on feasibility of the *average* link rates. It is important to recall that LP relaxations result in widening the feasibility space, i.e., the solutions obtained under the average rate (relaxed) constraints may be infeasible under the instantaneous rate constraints. However, since we seek to characterize the maximum throughput, these relaxations will only make the maximum less tight. There is a clear trade-off between the solution quality and the problem size/complexity. To keep the problem simple while drawing useful conclusions, we use the LP-relaxed constraints instead.

Let's consider a set of time slots $S \subseteq T$ of cardinality $\tau = |S|$, and define a continuous variable ρ_{ik} to be

$$\rho_{ik} = \frac{1}{\tau} \sum_{t \in S} y_{ik}^t, \quad \forall i \in L, \forall k \in K.$$

Note that ρ_{ik} represents the fraction of time in S during which link i is active on band k. Recall that this continuous variable is *averaged* over the length of the time slot set S. Hence, the longer S is, the more accurate this average becomes. Throughout the rest of the paper, we assume that the length of S is long enough for these variables to reflect accurate averages. By using this continuous variable, one can provide LP relaxations to the packet-level constraints described in Section 5.1. For example, by summing both sides of (1) over S and interchanging summations between k and t, one can obtain

$$\sum_{k \in K} \sum_{i \in L_m} \rho_{ik} \leq 1, \quad \forall m \in N. \tag{4}$$

When applying the same technique to the interference constraints, given by (2) and (3), one can obtain the following LP-relaxed interference constraints:

$$\begin{cases} (M - \vartheta_{r(i)} + 1)\rho_{ik} + \sum_{j \in C_i^-} \rho_{jk} \leq M, \\ (M - \theta_{t(i)} + 1)\rho_{ik} + \sum_{j \in C_i^+} \rho_{jk} \leq M, \end{cases} \tag{5}$$

for all $(i, k) \in L \times K$.

6 Maximum Multicommodity Flow

In this section, we first formulate the end-to-end multicommodity flow routing problem as a standard packing LP and then propose a fast solution algorithm for it.

6.1 Packing LP

Let's consider a multiband, multiantenna WMN routing instance that consists of a set Q of commodities. For every $q \in Q$, let P_q denote the set of all possible paths between $s(q)$ and $d(q)$—a possible path in P_q is a sequence of (link,band) pairs between $s(q)$ and $d(q)$. By letting x_p denote the rate of a path p, one can write

$$\rho_{ik} = \frac{1}{c_{ik}} \sum_{q \in Q} \sum_{p \in P_q : p \ni (i,k)} x_p,$$

for all $(i, k) \in L \times K$. Now, by replacing ρ_{ik} with the above expression in both the radio and interference constraints (4) and (5), the multicommodity flow routing problem can be formulated as a standard packing LP as shown in Table 1.

6.2 An Algorithm for Solving the Packing LP

We now propose a fast approximation algorithm for solving the packing LP. The idea is as follows: Instead of finding a solution to the packing LP problem, we propose an algorithm that finds a solution to its dual. The dual of the packing LP is shown in Table 2, and consists of finding weight assignments $u(m)$, $v(i, k)$, and $w(i, k)$ $\forall m \in N$ and for all pairs $(i, k) \in L \times K$ such that the sum of all weights is minimized while ensuring the shortest weighted path to be greater than unity. In matrix notation, the packing LP and its dual can, respectively, be written as $\max\{a^T x : Ax \leq b, x \geq 0\}$ and $\min\{b^T z : A^T z \geq a, z \geq 0\}$, where $a^T = [1, 1, \ldots, 1]$ is a vector of length $\sigma = \sum_{q \in Q} |P_q|$, $b^T = [1, 1, \ldots, 1]$ is a vector of length $\omega = |N| + 2 \times |K|$, and A is a $\omega \times \sigma$ matrix whose positive elements can be extracted from Table 1 or Table 2.

Our proposed approximation algorithm for solving the packing LP is given in Table 3. The algorithm follows from the work in [26]. Let ϵ be a fixed positive number and $\delta = (1 + \epsilon)[(1 + \epsilon)\omega]^{-\frac{1}{\epsilon}}$. The algorithm starts off by assigning δ to all weights and then proceeds iteratively. In each iteration, a *length* function $Z : L \times K \longrightarrow \Re^+$, which assigns each pair (i, k) the value $Z(i, k)$ (see Table 3 for the expression of $Z(i, k)$), is determined. The algorithm then

TABLE 1
Primal Packing LP Problem

Maximize $\sum_{q\in Q}\sum_{p\in P_q} x_p$ **subject to:**

$$\sum_{i\in L_m}\sum_{k\in K}\frac{\sum_{q\in Q}\sum_{p\in P_q:p\ni(i,k)} x_p}{c_{ik}} \leq 1, \quad \forall m \in N$$

$$(M-\theta_{t(i)}+1)\frac{\sum_{q\in Q}\sum_{p\in P_q:p\ni(i,k)} x_p}{Mc_{ik}} + \sum_{j\in C_i^+}\frac{\sum_{q\in Q}\sum_{p\in P_q:p\ni(j,k)} x_p}{Mc_{jk}} \leq 1, \quad \forall (i,k)\in L\times K$$

$$(M-\vartheta_{r(i)}+1)\frac{\sum_{q\in Q}\sum_{p\in P_q:p\ni(i,k)} x_p}{Mc_{ik}} + \sum_{j\in C_i^-}\frac{\sum_{q\in Q}\sum_{p\in P_q:p\ni(j,k)} x_p}{Mc_{jk}} \leq 1, \quad \forall (i,k)\in L\times K$$

$$x_p \geq 0, \quad \forall p\in P_q, \forall q\in Q$$

TABLE 2
Dual Packing LP Problem

Minimize $\sum_{m\in N} u(m) + \sum_{(i,k)\in L\times K} v(i,k) + \sum_{(i,k)\in L\times K} w(i,k)$ **subject to:**

$$\sum_{(i,k)\in p}\left\{\frac{u(t(i))}{c_{ik}} + \frac{u(r(i))}{c_{ik}} + \frac{M-\theta_{t(i)}+1}{Mc_{ik}}v(i,k) + \sum_{j\in C_i^+}\frac{v(j,k)}{Mc_{jk}} + \frac{M-\vartheta_{r(i)}+1}{Mc_{ik}}w(i,k) + \sum_{j\in C_i^-}\frac{w(j,k)}{Mc_{jk}}\right\} \geq 1, \forall p\in P_q, \forall q\in Q$$

$$u(m), v(i,k), w(i,k) \geq 0, \forall m\in N, \forall i\in L, \forall k\in K$$

computes the shortest weighted path among all pairs $(s(q), d(q)), \forall q \in Q$, where a path between a (source,destination) pair, $(s(q), d(q))$, is a set of (link,band) pairs that connects the source to its destination. A flow is then routed via this shortest path. The rate of this flow is chosen such that the minimum capacity edge belonging to the shortest path is saturated; the capacity of an edge e belonging to the shortest path p is $A(e,p)$. The weights of (link,band) pairs belonging to this path are increased as a result of this flow. The algorithm terminates when the sum of all weights is greater than or equal to unity.

Given $\epsilon > 0$, the proposed algorithm finds a $(1-\epsilon)^{-2}$-approximation to the multicommodity flow optimal solution in running time that depends polynomially on ϵ^{-1}. The input parameter ϵ can be appropriately chosen so that a solution with acceptable quality is obtainable in polynomial time (trading off some precision for faster execution). The following theorem states the trade-off between the solution accuracy and the running time of the algorithm. The proof follows from [26]:

Theorem 1. *For any fixed* $\epsilon, 0 < \epsilon < 1$, *the proposed algorithm, shown in Table 3, finds a throughput solution* $\hat{\eta}$ *to the packing LP, described in Table 1, that 1) satisfies* $(1-\epsilon)^2\eta^* \leq \hat{\eta} \leq \eta^*$, *where* η^* *is the optimal solution, and 2) completes in* $\omega\lceil\frac{1}{\epsilon}log_{1+\epsilon}\omega\rceil \times \mathcal{T}$, *where* $\mathcal{T}$ *is the time needed to compute the shortest path.*

7 Evaluation

In this section, we first identify and analyze the maximum achievable throughput of multiband, multiantenna WMNs by using the proposed algorithm for many randomly generated network instances. We then show how the thus-obtained results and analysis can be used by designers to determine the optimal parameters that maximize the overall throughput of WMNs.

It is important to recall that our goal is to evaluate and identify MIMO's potential in terms of its spatial reuse (*not*

TABLE 3
Approximation Algorithm

Initialize:
$u(m) = v(i,k) = w(i,k) = \delta, \forall m \in N, \forall (i,k) \in L\times K$
$f = 0$
While $(\sum_{m\in N} u(m) + \sum_{(i,k)\in L\times K}[v(i,k) + w(i,k)] < 1)$

- Assign each pair $(i,k)\in L\times K$ the number $Z(i,k) = \frac{u(t(i))}{c_{ik}} + \frac{u(r(i))}{c_{ik}} + \frac{M-\theta_{t(i)}+1}{Mc_{ik}}v(i,k) + \sum_{j\in C_i^+}\frac{v(j,k)}{Mc_{jk}} + \frac{M-\vartheta_{r(i)}+1}{Mc_{ik}}w(i,k) + \sum_{j\in C_i^-}\frac{w(j,k)}{Mc_{jk}}$.
- Find the shortest weighted path p^* among all paths between $s(q)$ and $d(q)$ for all $q\in Q$. Let l^* and n^* be the sets of all (i,k) and all nodes forming p^*.
- Write the expression $\sum_{(i,k)\in l^*} Z(i,k)$ in the form $\sum_{m\in n^*}\lambda_m u(m) + \sum_{(i,k)\in l^*}[\mu_{ik}v(i,k) + \nu_{ik}w(i,k)]$. Let $r^* = \max_{m\in n^*,(i,k)\in p^*}\{\lambda_m, \mu_{ik}, \nu_{ik}\}$.
- Assign:
  $u(m) \longleftarrow u(m)(1+\epsilon\frac{\lambda_m}{r^*}), \forall m\in n^*$
  $v(i,k) \longleftarrow v(i,k)(1+\epsilon\frac{\mu_{ik}}{r^*}), \forall (i,k)\in p^*$
  $w(i,k) \longleftarrow w(i,k)(1+\epsilon\frac{\nu_{ik}}{r^*}), \forall (i,k)\in p^*$
  $f \longleftarrow f + \frac{1}{r^*}$

EndWhile
Compute approximated throughput: $\hat{\eta} = \frac{f\epsilon}{1+log_{1+\epsilon}\omega}$

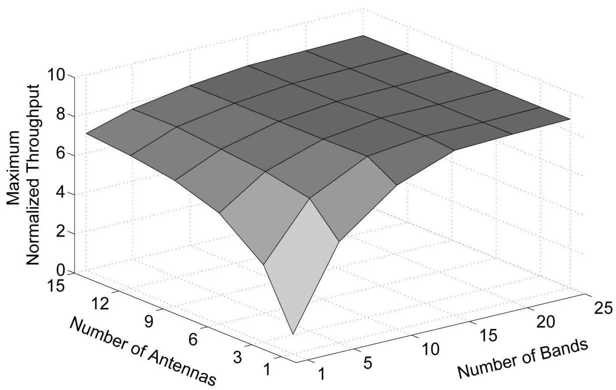

Fig. 3. The maximum throughput. $|N| = 50$, $|Q| = 25$, $d = 16$ m.

multiplexing) capabilities. Hence, throughput behaviors and analysis presented in this section are a consequence of spatial reuse only.

7.1 Parameter Setting

We randomly generate WMNs, each consisting of $|N|$ nodes, each of which is equipped with an antenna array of π elements. Nodes are uniformally distributed in a cell of size 100 m $\times$ 100 m, where two nodes are considered neighbors if the distance between them does not exceed d m (i.e., communication range). We assume that $c_{ik} = 1$ for all $(i, k) \in L \times K$. For each random WMN, $|Q|$ (source,destination) pairs are randomly generated to form $|Q|$ multicommodity flows.

Our proposed approximation algorithm is solved for each WMN to find the maximum achievable throughput by the $|Q|$ commodity flows. The approximation parameter ϵ is set to 0.05. Hence, the approximated solutions, computed using the approximation algorithm, are found to be within 10 percent of their exact values. All data points in all figures represent averages over all of the generated WMNs. For every simulation scenario, we keep generating graphs and solving them until the measured average throughput converges to within 5 percent of its real value at a 98-percent confidence interval. This means that with probability 0.98, the plotted/measured average throughput for each simulation scenario falls within 5 percent of the real/unknown average throughput.

7.2 Asymptotic Throughput Behavior

Fig. 3 shows the maximum achievable normalized[1] throughput as a function of the number of antennas and/or the number of bands. Note that as the number of antennas and/or bands increases, the maximum achievable throughput first rises and then flattens out asymptotically. Let's, for example, consider the case when the number of bands equals 1. Augmenting the number of antennas from 1 to 6 increases the normalized network throughput by a factor of 5.6 (from 1 to 5.6), whereas augmenting it from 6 to 12 increases the network throughput by only a factor of approximately 1.1 (from 5.6 to 6.7); the normalized network throughput is bounded by a factor of 7 as the number of antennas increases. A similar behavior is observed when the number of bands is increased from 1 to 25 while fixing the number of antennas. Recall that multiple bands and/or multiple antennas are capable of increasing the network throughput by allowing multiple communications to occur simultaneously in the same vicinity. For instance, multi-antenna-equipped nodes can use their antennas to suppress undesired signals sent by nearby transmitters, allowing them to receive interference-free signals concurrently with nearby transmitted signals. Likewise, multiband-capable nodes can choose and switch to idle spectrum bands, also allowing them to avoid interference with nearby signals. Intuitively, it can then be concluded that the more antennas and/or spectrum bands a node can use, the more nearby transmitters' signals can be nulled, and hence, the higher the achievable network throughput. However, because nodes of a given network have a fixed number of interfering nodes, increasing the number of antennas and/or bands beyond that of a node's fixed number of interfering nodes can no longer increase the throughput of the network. This explains the asymptotic upper bound on the maximum throughput as a function of the number of antennas and/or bands.

Another point to note is that for a high number of antennas (bands), the maximum achievable throughput remains unchanged regardless of the number of bands (antennas). This is because when the number of antennas is large enough, all sessions can be active at the same time even when each of them is allowed to communicate on one band only. Likewise, when the number of bands is large, multiple sessions can also be running concurrently, each on a separate band even when each node is equipped with a single antenna system.

In summary, given a WMN (i.e., defined by its node degree, connectivity, transmission range, etc.) and the number of bands that nodes are allowed to communicate on, there is an optimal number of antennas beyond which multiple antennas can no longer increase the network throughput. Likewise, given a WMN and a number of antennas, there is an optimal number of spectrum bands beyond which the network throughput can no longer be increased with additional bands. Next, we will show how sensitive such optimal numbers are to network parameters, such as transmission range and node degree.

7.3 Effects of Transmission Range/Power

We now study the effects of the transmission range on the maximum achievable throughput of multiband, multi-antenna WMNs. Recall that the greater the transmission range, the more the interference, but also the higher the node degree. While a higher node degree usually yields a more network throughput, more interference results in a lesser throughput. We would then like to study the extent to which, if any, such a trend holds when WMNs are both multiband capable and multiantenna equipped.

Fig. 4 shows the maximum achievable throughput as a function of both the transmission range and the number of spectrum bands when the number of antennas is 1 (Fig. 4a), 6 (Fig. 4b), and 12 (Fig. 4c). Throughout this section, we set the number of nodes $|N|$ to 50 and the number of multicommodity flows $|Q|$ to 25, and vary the transmission

1. Normalized w.r.t. the achievable throughput when nodes are each equipped with one antenna and allowed to use one spectrum band only.

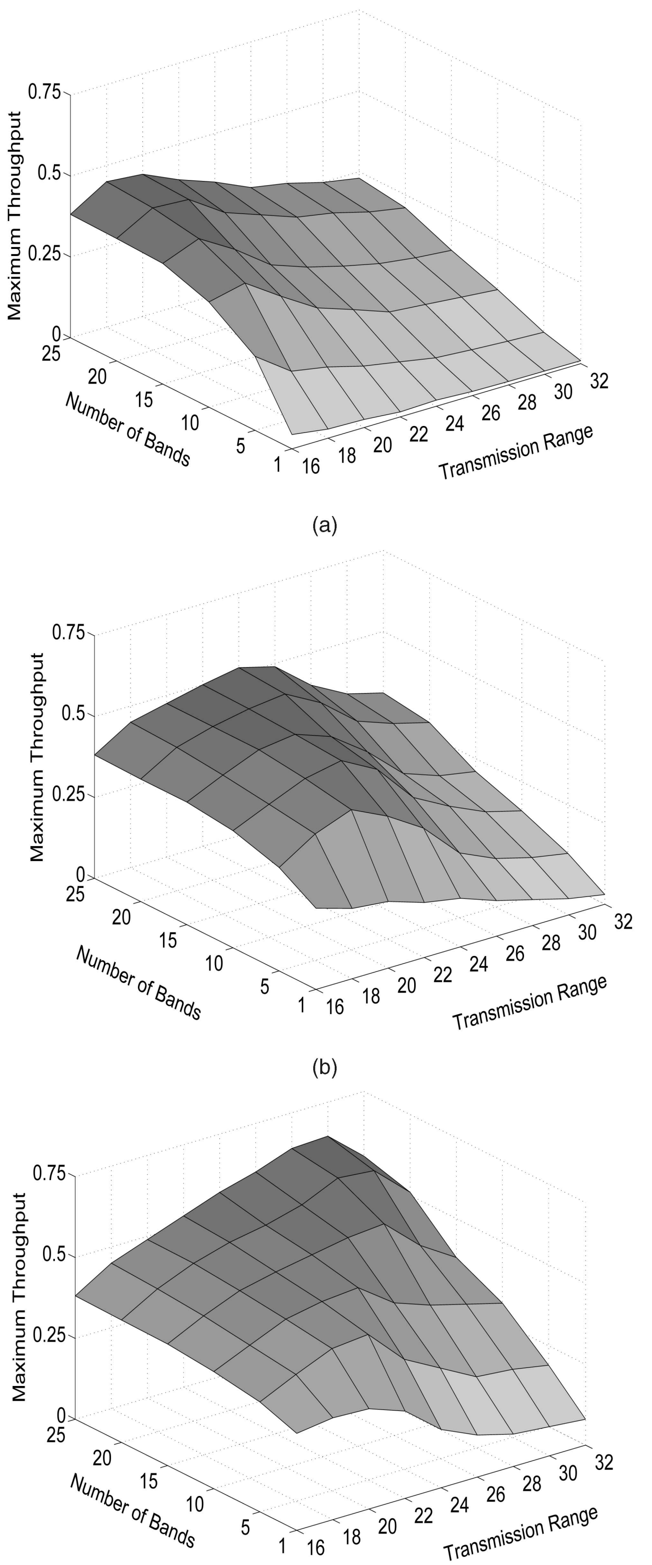

Fig. 4. Effect of transmission range on throughput. $|N| = 50$, $|Q| = 25$. (a) Number of antennas $= 1$. (b) Number of antennas $= 6$. (c) Number of antennas $= 12$.

range d from 16 to 32 m, the number of bands from 1 to 25, and the number of antennas from 1 to 12. There are two important and useful trends to observe from the obtained results as discussed below.

7.3.1 Transmission Range/Power Optimality

Note that irrespective of the number of bands and/or the number of antennas, as the transmission range increases, the overall throughput keeps increasing until it reaches an optimal value after which it starts decreasing. In other words, for each combination of the number of bands and the number of antennas, there is an optimal transmission range at which the overall network throughput is maximized. Recall that the longer a node's transmission range, the more neighbors the node is likely to have. While a longer transmission range enables nodes to have more paths to route their traffic through, it also generates more interference for them to combat. On the other hand, shorter transmission ranges yield lesser interference, but also lesser path diversity. Therefore, when the transmission range is too short, although the resulting interference is relatively low, it is the lack of path diversity that limits the achievable throughput of WMNs despite their multiband and multiantenna capabilities. On the other hand, when the transmission ranges are too long, the interference dominates, thereby limiting the throughput. In this case, the multiband and multiantenna capabilities are not sufficient enough to suppress the extra interference caused by the long reach of transmitted signals.

When the transmission ranges are appropriately chosen (neither too short nor too long), nodes can take advantage of the increased number of paths to find better routes while effectively combating the interference by using their multiband and multiantenna capabilities. In such a case, the throughput will be increased as more concurrent communication sessions are enabled in the same vicinity. This explains the convex behavior of the throughput as a function of the transmission range.

7.3.2 Transmission Range/Power Sensitivity

For any given number of antennas, the results show that the optimal transmission range at which the overall network throughput is maximized keeps increasing as the number of spectrum bands increases. For example, when the number of antennas is 6 (Fig. 4b), the optimal transmission range is found to be 20 when the number of bands is 5, whereas it is 24 when the number of bands is 20. A similar behavior is observed when the number of antennas is varied with the number of bands fixed. The optimal transmission range also increases with the number of antennas for any given number of allowed spectrum bands.

Recall that the multiband and multiantenna capabilities enhance the overall throughput of WMNs by allowing multiple concurrent communication sessions in the same vicinity. Hence, the more of these capabilities a WMN is empowered with, the more concurrent communication sessions it can allow, and hence, the higher the overall throughput it can achieve. However, providing a WMN with more capabilities than what could possibly be achieved in terms of number of concurrent sessions does not increase the overall network throughput. The number of possible concurrent communication sessions for enhancing network throughput is determined by the number of neighbors the concerned nodes interfere with, which, in turn, is determined by the transmission range. As we discussed before, a longer transmission range corresponds to more possible concurrent sessions through higher path diversity. This explains why the higher the multiband and/or multiantenna capabilities a WMN is provided with, the longer the transmission range at which the overall network throughput is maximized, i.e., the higher the optimal transmission range/power.

It is worth mentioning that while a greater transmission range provides nodes with higher path diversity, it also shortens the average path length of flows as well as it provides nodes with more interference to deal with (as the average number of neighbors also increases as a result of increasing the transmission range). Therefore, when transmission ranges are long and the number of antennas is small, interference dominates as these antennas may not be enough to combat the extra interference caused by the long ranges of transmission, thereby achieving less overall throughput. When the number of antennas is large enough, nodes can, however, take advantage of the increased number of paths to find better routes while effectively combating the interference by using their antennas. In this case, the throughput will increase as more concurrent communication sessions are enabled in the same vicinity. Thus, for a large number of antennas, the achievable throughputs for long transmission ranges are greater than those for short transmission ranges.

The results of the transmission range study can be summarized as follows: For every combination of the number of antennas, the number of accessible spectrum bands, and the number of mesh nodes, there is an optimal transmission range (or transmission power) that maximizes the overall achievable throughput of the WMN. In Section 7.5, we use this study to derive guidelines for network designers to determine the optimal transmission ranges of WMNs given the other parameters.

7.4 Effect of Node Degree/Density

We now study the effect of the node degree on the maximum achievable throughput. The node degree, defined as a node's number of neighbors, can be changed by varying the transmission range and/or the node density. The higher the transmission range and/or the node density, the greater the node degree, and vice versa. As illustrated in Section 7.3, an increase in the transmission range causes more interference. However, an increase in the node density does not increase interference (provided the number of flows $|Q|$ is kept the same). To decouple the effect of node degree from that of interference, we, therefore, use node density as a way of varying the node degree.

In this study, we fix the transmission range d at 30 and the number of commodity flows $|Q|$ at 25, and vary the average node degree from 4 to 10 by varying the node density from 0.2 percent ($|N| = 20$) to 0.5 percent ($|N| = 50$). In Fig. 5, we show the maximum achievable throughput as a function of both the node degree and the number of bands when the number of antennas is 1 (Fig. 5a), 6 (Fig. 5b), and 12 (Fig. 5c). We make two observations regarding the effect of node degree/density on the achievable throughput as described below.

7.4.1 Node Degree/Density Optimality

As shown in Fig. 5, regardless of the number of bands and/or antennas, as the average node degree increases, the overall throughput first increases, then flattens out, and remains unchanged. That is, for each combination of the number of bands and the number of antennas, there is a node degree threshold beyond which the overall achievable

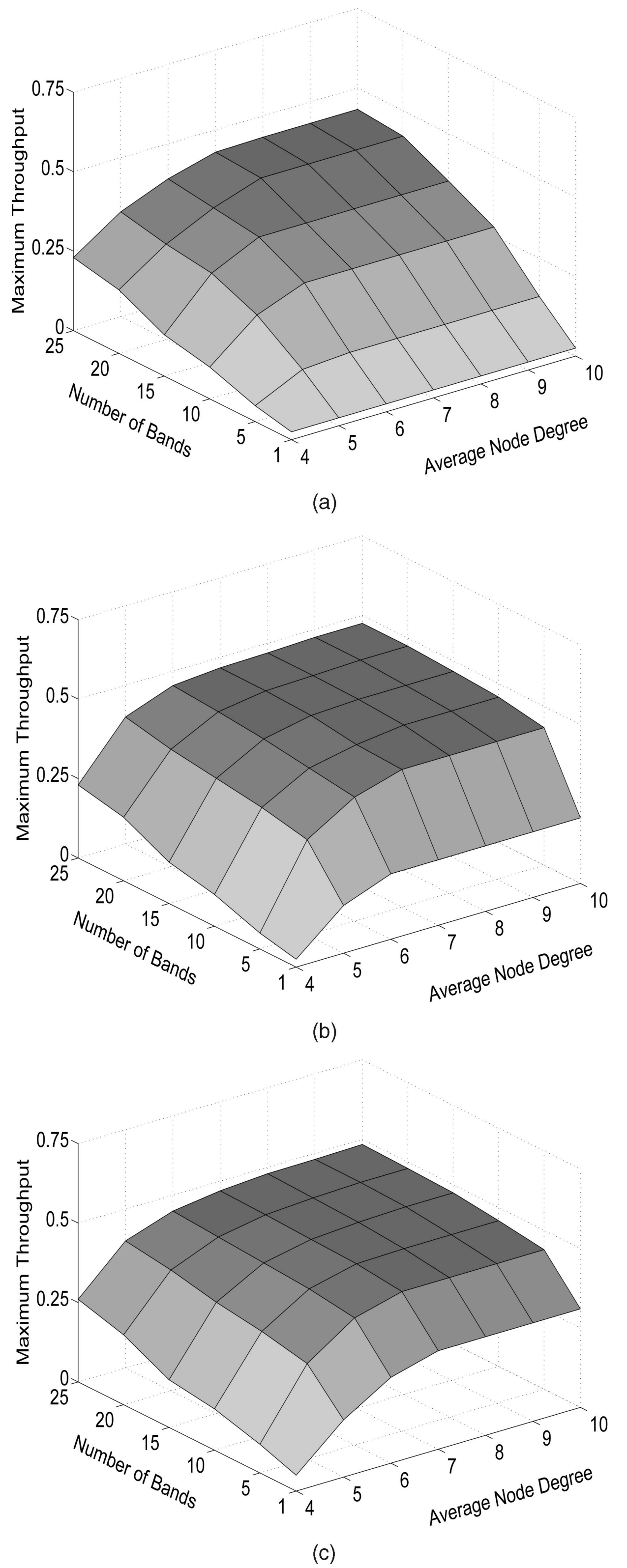

Fig. 5. Effect of node degree on throughput. $d = 30$, $|Q| = 25$. (a) Number of antennas $= 1$. (b) Number of antennas $= 6$. (c) Number of antennas $= 12$.

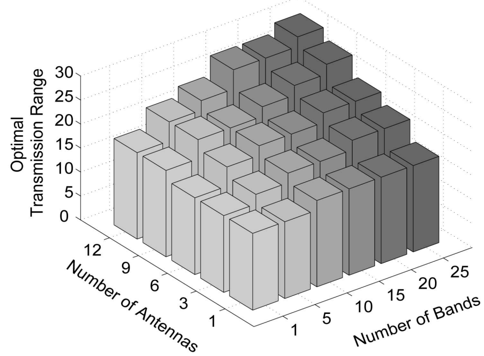

Fig. 6. The optimal transmission range as a function of number of bands and number of antennas. $|N| = 50$, $|Q| = 25$.

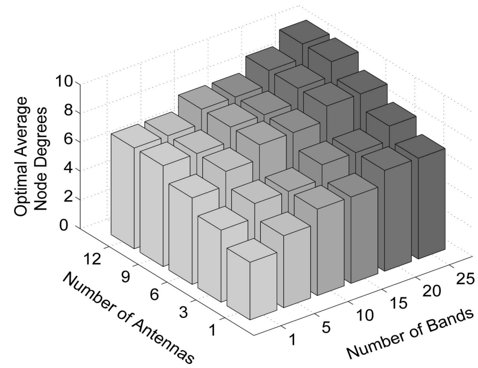

Fig. 7. The optimal average node degree as a function of number of bands and number of antennas. $d = 30$ m, $|Q| = 25$.

network throughput can no longer be improved with additional nodes.

As discussed above, increasing the node degree through node density increases path diversity, but not the interference. Therefore, the network throughput can only be increased by increasing the node degree, which explains the monotonic behavior of the throughput as a function of the node degree. For a given number of antennas and bands (i.e., for given multiband and multiantenna capabilities), the higher the node degree, the more paths are available for routing, and hence, the more throughput the network can achieve. Note that the increase in throughput is a consequence of exploiting the multiband and multiantenna capabilities for path diversity. Hence, the network throughput can no longer be increased when the limit of these capabilities is reached, explaining the asymptotic behavior of the throughput as a function of the node degree.

7.4.2 Sensitivity of Node Degree/Density

Observations similar to those made in the case of transmission range are also made in the case of node degree. Irrespective of the number antennas, the optimal average node degree is observed to increase as the number of spectrum bands increases. Similarly, the optimal average node degree increases with the number of antennas, regardless of the number of allowed spectrum bands. For instance, when the number of antennas is 6 (Fig. 5b), the optimal average node degrees are 7 and 9 when the number of bands is 5 and 20, respectively. The more antennas and/or spectrum bands nodes can use, the more path diversity can be exploited, and hence, the higher the optimal node degree/density.

The results of studying the average node degree can be summarized as follows: For every combination of the number of antennas, the number of accessible spectrum bands, and the transmission range, there is an optimal node degree (or node density) that maximizes the overall throughput of WMNs. Then, we derive guidelines for determining the optimal node degree of WMNs based on this study.

7.5 Design Guidelines

We now demonstrate how our results can be used to derive guidelines for designing WMNs that are multiband capable and multiantenna equipped. The thus-derived guidelines allow network designers to determine the optimal network parameters, such as the transmission range (or transmission power) and the node degree (or node density), which maximize the overall throughput of next-generation WMNs.

For the purpose of illustration, we consider WMNs, each of which consists of 50 mesh nodes deployed in an area of $100 \times 100\ \mathrm{m}^2$, and assume that there are $|Q| = 25$ end-to-end multihop flows in the WMN. We extend the simulation scenarios of Section 7.3 to include more combinations of numbers of antennas and spectrum bands. Our results, showing the optimal transmission range as a function of the number of antennas and the number of spectrum bands, are summarized in Fig. 6. Knowing the number of antennas each mesh node is equipped with, and the number of bands the WMN is allowed to communicate on, a network designer can use this figure to determine the transmission range so that the overall network throughput is maximized. The figure, for example, shows that the optimal transmission range of a WMN, whose nodes are each equipped with six antennas and allowed to communicate on 10 spectrum bands, is 22 m. There are two points to mention about these results. First, because the transmission range is often controlled by means of transmission power, these guidelines can also be regarded as a way of determining the optimal transmission power. Second, although for the sake of illustration, we considered 50 mesh nodes, one can use the proposed approximation algorithm to derive similar design guidelines for WMNs with a different number of nodes.

Our framework can also be used to optimize other WMN parameters. For example, if the transmission range/power is fixed a priori and cannot be changed, then we can still optimize other network parameters. Let's fix the transmission range at 30 m and the area in which the nodes are to be deployed to $100 \times 100\ \mathrm{m}^2$. We can now, for example, determine the optimal node degree/density at which the overall network throughput is maximized. For different combinations of numbers of antennas and spectrum bands, we use the proposed algorithm for various node degrees to maximize the network throughput. The results, showing the optimal node degree for each combination of numbers of antennas and bands, are plotted in Fig. 7. For example, when the transmission range is 30 m, the number of antennas is 6, and the number of allowed spectrum bands is 10, the optimal average node degree is about 8 (corresponding to 40 nodes).

There are two important points that require attention. First, even though we considered optimization of the transmission range and the node density, one can also use this framework to optimize other network parameters, such as the type and condition of traffic, and the hop count of multihop flows. Second, although the optimization is based on a network-layer metric (i.e., the multihop achievable throughput), it implicitly considers cross-layer (MAC and PHY) coupling effects as well. These effects are accounted for through the cross-layer modeling in Section 4.1.

In summary, the framework developed in this paper serves as a basis for deriving design guidelines for next-generation WMNs. This method is flexible and fast. It is flexible because it can be used to optimize various WMN parameters, provided the other parameters are known a priori. The proposed approximation algorithm is also fast; the input parameter ϵ can be so chosen that a solution to the WMN routing optimization problem with acceptable accuracy can be obtained in a reasonable amount of time. For example, an approximate solution, within 10 percent of its exact value, can be found in several minutes by using the proposed algorithm (for $\epsilon = 0.05$); whereas, it can take many hours to solve with traditional linear programming methods.

8 Discussion

The focus of this work is on characterizing achievable throughput of multihop wireless networks when they are multiantenna, multichannel capable, but single-radio equipped only. Throughput performance of multichannel networks with multiradio capabilities has also been investigated by several researchers (e.g., [13], [20]), and having multiple radios is shown to increase achievable throughput [13]. Now, the natural question is: how would the throughput behave when we consider the three dimensions—multiantennas, multichannels, and multiradios—all together? Although considering all three dimensions at the same time makes network throughput even more challenging to characterize, and hence, too difficult to predict its behavior without careful modeling, we will provide a quantitative discussion on the subject (an accurate and complete characterization of throughput in such networks will be worth a separate paper, which is part of our future work).

Suppose that there are a antennas, c channels, and r radios (we will henceforth use the notation (a, c, r)-networks to mean networks with a antennas, c channels, and r radios). Let us assume that a radio can only be used to communicate on one channel at a time. (Of course, radios change channels from one time slot to another). Roughly, we can say that the total achievable throughput of (a, c, r)-networks should be at least r times as much as that of $(\frac{a}{r}, 1, 1)$-networks. Here, we simply assumed that the number of antennas is split equally among all r radios, and that each channel is used by all nodes, each equipped with $\frac{a}{r}$ antennas and 1 radio. The throughput can, however, be higher than r times that of $(\frac{a}{r}, 1, 1)$-networks for two reasons. The first reason is antenna-allocation flexibility. The number of antennas at each node does not have to be split equally among all radios. For example, radios with high contention may be assigned more antennas. Such flexibility may lead to higher upper bounds on throughput. The second reason is channel-allocation flexibility (assume $c > r$). r channels among all c channels can be assigned to the r radios. This may lead to more relaxed constraints, which may, in turn, lead to higher achievable throughput. Another observation that we can also make is that the impact of the number r of radios depends on the number c of available channels as there is a one-to-one mapping between r and c (a channel cannot have more than one radio at a given time). This is, however, not the case for the number of antennas vis-a-vis of the number of radios, i.e., the allocation of the number of antennas across different radios, and hence, across different channels is more flexible as there can be many-to-one mappings.

9 Conclusion and Future Work

We proposed a framework that can be used to 1) identify the limits and potential of SDRs and MIMO technologies in terms of the maximum throughput that they can provide to WMNs and 2) derive guidelines for designing and optimizing multiband-capable, multiantenna-equipped WMNs.

While SDRs are used in this study as a means of enabling WMNs with dynamic and adaptive multiband access, MIMO is used as a means of increasing the spatial reuse of spectrum, and hence, the total network throughput. It is, however, important to note that MIMO can be exploited to augment network throughput not only via spatial reuse but also via spatial division multiplexing. In the future, we intend to investigate and characterize the total throughput that multiband, multiantenna WMNs can achieve when MIMO benefits are exploited for spatial multiplexing.

Acknowledgments

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