Admission and Eviction Control of Cognitive Radio Users at Wi-Fi 2.0 Hotspots

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Abstract—Cognitive radio (CR)-based Wi-Fi 2.0 hotspots are introduced as an attractive application of dynamic spectrum access (DSA), at which a wireless service provider (WSP) leases licensed channels via secondary market and offers Internet access to CR-enabled customers by opportunistically utilizing the leased spectrum. The CR users access the channels only when they are temporarily unoccupied by their legacy users, and pay a usage charge according to the WSP's pricing policy. In this paper, we study the profit maximization problem of a WSP by deriving the (sub)optimal control of *admission* (at CR user arrivals) and *eviction* (upon return of the legacy users) of CR users. We formulate the problem as a semi-Markov decision process (SMDP) with two quality-of-service (QoS) constraints on arrival-blocking and service-dropping probabilities, which is solved by the linear programming techniques. Using an extensive numerical analysis, we show that the derived policy achieves up to 22.5-44 percent more profit than simple complete-sharing algorithms in the tested scenarios. In addition, we evaluate the impact of the number of leased channels and pricing on the achieved profit, and study the tradeoffs between the two QoS constraints.

Index Terms—Wi-Fi 2.0, admission and eviction control, cognitive radio (CR), CR hotspot, spectrum opportunity

1 Introduction

Dynamic Spectrum Access (DSA) has opened a new way of solving the spectrum-scarcity problem caused by the conventional *static* spectrum-allocation policy [2]. DSA paves ways to enhance spectrum utilization of the legacy spectrum resources by enabling unlicensed secondary users (SUs) to opportunistically utilize the *whitespaces* of the licensed spectrum where it is left unused by legacy primary users (PUs). SUs are also called Cognitive Radio (CR) users since they are equipped with CR devices to dynamically identify the time-varying spectrum availability due to the spectrum access patterns of PUs.

The application of DSA ranges from public to commercial and military networks. In this paper, we focus on a commercial DSA application, *Wi-Fi 2.0* [3], [4], that refers to Wi-Fi-like service using whitespaces.[1] Wi-Fi 2.0 has been identified as an important step in DSA development due to its similarity to today's Wi-Fi and its superior service quality such as larger coverage and the wall-penetrating ability [6] thanks to the more favorable propagation characteristics of the licensed spectrum (e.g., TV bands). Note that Wi-Fi over whitespaces is not necessarily based on the same type of protocol as IEEE 802.11 [7].

1. Wi-Fi 2.0 is also referred to as Wi-Fi on Steroids, Super Wi-Fi, and WhiteFi [5].

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Wi-Fi 2.0 service is operated by a CR wireless service provider (WSP) who dynamically leases licensed spectrum bands from the spectrum license holders (or licensees) and opportunistically utilizes them to provide the service at a *CR Wi-Fi hotspot* (henceforth simply referred to as a CR hotspot) to CR-enabled customer terminals. The spectrum leasing is performed in the dynamic spectrum market (DSM) [8] where the spectrum broker (SB) auctions off the licensees' spectrum bands to the CR WSPs. The SB is either the regulatory authorities (e.g., FCC in US and Ofcom in United Kingdom) or an authorized third party. The WSPs compete with each other to lease as many spectrum bands as necessary which will be opportunistically utilized at their CR hotspots.[2] The interactions in Wi-Fi 2.0 is illustrated in Fig. 1.

1.1 Contributions

The contribution of this paper is twofold. First, we propose a new spectrum reuse model called *preemptive spectrum lease* which is one realization of the private commons model introduced in [9], as illustrated in Fig. 2. In our model, the license holders temporarily lease their channels to CR WSPs via *periodic* dynamic spectrum auction (e.g., hourly) and charge them for their opportunistic use of paid-but-idle channels. The WSPs are allowed to use the leased channels only when they are temporarily unoccupied by the PUs because the licensed users are given priority over the unlicensed CR users. Therefore, the CR users must vacate a channel to which PUs return (called *channel vacation*) where the channel state changes from "available to SUs" to "occupied by PUs," and should utilize the remaining idle channels afterwards. When PUs no longer transmit on the vacated channel, it can be used again by the SUs. Once a leasing term ends, the leased channels are all returned to

2. Note that licensees indicate PUs and the SB is just an auctioneer who helps the PUs lease their spectrum to the WSPs who are classified as SUs.

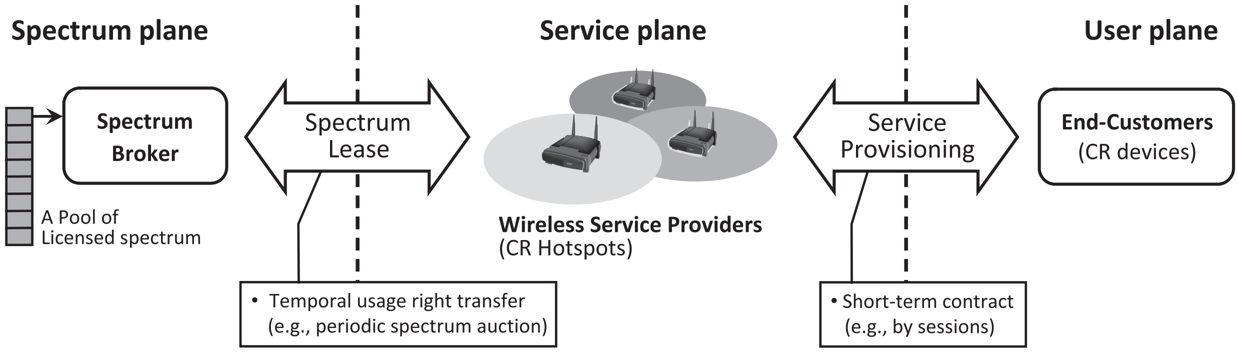


Fig. 1. Interplane interactions in Wi-Fi 2.0.

the licensees and the WSPs must reparticipate in the auction to lease new channels.

Next, we solve the profit maximization problem of a WSP by optimizing two types of user control: admission and eviction. Admission control determines if a newly arriving customer should be admitted to or rejected from the service, to achieve better profit. Although optimal user admission control in isolation was studied in [10] for the case of static spectrum availability, CR networks face a unique challenge—time-varying spectrum availability due to PUs' activities—that necessitates joint control of user admission and eviction. At channel vacation, the customers previously assigned to the channel (called *in-service users*) have to be relocated by the WSP to the other remaining idle channels. However, in case the remaining idle channels cannot fully support spectrum demands of all in-service users,[3] the WSP should determine which users to be evicted from its network. The evicted users will be compensated with some form of reimbursement, which may differ by the user-specific spectrum demands and thus affect the WSP's profit.

To derive the optimal user admission and eviction controls, we first model it as a semi-Markov decision process (SMDP) and a linear programming (LP) algorithm is proposed to derive the solution. The solution derived from the SMDP formulation becomes optimal for channels with exponential ON/OFF periods, and a suboptimal solution for generally distributed ON/OFF periods is also derived via approximation of the distribution of channel state transition. QoS provisioning for CR end-users is also considered by adding two constraints to the LP algorithm: the probability of blocking newly arriving users and the probability of evicting/dropping in-service users, so that the WSP can strike a balance between profit maximization and customer satisfaction.

1.2 Organization

Section 2 overviews related work, and then Section 3 introduces the system model and the basic assumptions used throughout this paper. In Section 4, the problem of maximizing a WSP's profit is modeled as an SMDP and its relevant components are derived. Section 5 presents an LP-based SMDP algorithm to determine the (sub)optimal user admission and eviction policies with constraints on the blocking and dropping probabilities. Section 6 introduces a prioritized multiclass service at CR hotspots and derives its optimal user controls. The proposed scheme is evaluated in Section 7 via numerical analysis and in-depth simulation, and the paper concludes with Section 8.

2 Related Work

Mutlu et al. [11] studied how to maximize a WSP's average profit, focusing on an optimal pricing policy without considering user admission control which could increase the WSP's profit further. Moreover, they assumed that PUs and SUs can simultaneously access the same channel, thus unneeding user eviction control, which is not possible if PUs are given priority over SUs. Ishibashi et al. [12] considered multihomed PUs, where each PU is either conventional or CR-enabled. They investigated enhancement of resource utilization with cognitive PUs switching between channels, and derived the blocking and dropping probabilities in such a scenario. However, no priority in channel access is given to the conventional PUs, thus unaccounting for user eviction. Wang et al. [13] proposed a primary-prioritized Markov approach where PUs have exclusive rights to access their own channels. Although they considered giving priority to PUs, user admission control was ignored and only one channel and two SUs were considered, thus limiting its applicability. Ross and Tsang [10] investigated the problem of optimal admission control on the users with different spectrum demands. However, their problem was limited to the case when channels are always available, thus unneeding user eviction control.

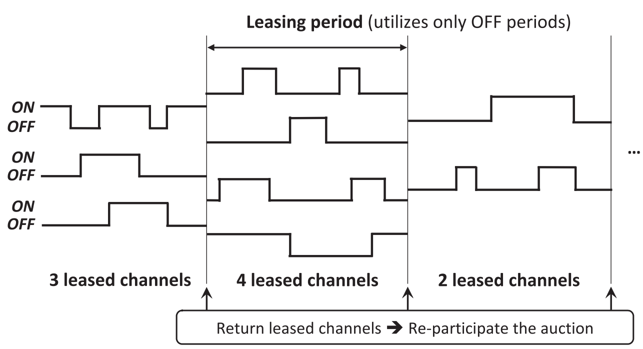


Fig. 2. The preemptive spectrum lease model: each channel has either ON or OFF states at a certain time where an ON period implies the channel is occupied by its PUs and an OFF period implies a whitespace.

3. QoS degradation (i.e., assigning less bandwidth than a user requested) is not considered in this paper, which is our future work.

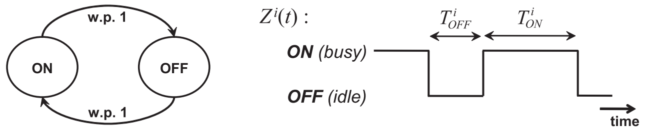


Fig. 3. Channel model: alternating renewal process with ON and OFF states.

A preliminary version of our ideas was presented in [1] where we assumed exponentially distributed ON (busy) and OFF (idle) periods and considered unprioritized user classes only. In this paper, we extend the results in [1] to generally distributed ON-OFF periods, and also derive optimal user controls for both unprioritized and prioritized multiclass services at CR hotspots.

3 System Model

In this section, we introduce the system model and assumptions to be used throughout this paper.

3.1 Channel Model

A channel is modeled as an ON/OFF alternating renewal process [14] as illustrated in Fig. 3, which has been frequently used in many applications [15], [16], [17].[4] An ON (or OFF) period implies the duration with (or without) PUs' signal activities whose sojourn time is represented by a random variable T^i_{ON} (or T^i_{OFF}) with the probability density function (pdf) of $f_{T^i_{ON}}(t)$ (or $f_{T^i_{OFF}}(t)$), $t > 0$, where $f_{T^i_{ON}}(t)$ and $f_{T^i_{OFF}}(t)$ can be *any* distribution function. ON and OFF states are statistically independent of each other. Note that we reserve i as the index of channel.

It is assumed that the transitions between ON and OFF states can be detected by either spectrum sensing or a PU signaling mechanism. First, spectrum sensing is a process of sampling the channel state (i.e., ON or OFF) to identify spectrum whitespaces, which has been studied extensively in the CR literature. It is further categorized as out-of-band and in-band sensing, where out-of-band sensing is used to discover whitespaces and in-band sensing is used to detect the OFF to ON transition of currently utilized idle channels. Since spectrum sensing is not the focus of this paper, we recommend interested readers to refer to [17], [18] for more details on spectrum sensing. Second, the PU signaling mechanism is a method with which the licensee indicates the presence/absence of the PUs in its channels so that the lessee can notice the status of the leased channels. Since the licensee can achieve extra profit via spectrum leasing, it is reasonable to assume that the licensee may be willing to build such an auxiliary mechanism to entice more CR WSPs to the secondary market and to protect its PUs more effectively.

Let C_i denote the capacity (or bandwidth) of channel i. In this paper, we assume channel capacities are homogeneous, i.e., $C_i = C$, $\forall i$, for ease of presentation. However, our SMDP model can be easily extended to the case of heterogeneous channel capacities at the expense of increased state/action spaces. With M (possibly noncontiguous) leased channels, a WSP's instantaneous capacity is given as $m \cdot C$, $0 \leq m \leq M$, according to the ON/OFF channel-usage patterns, where m is the number of idle channels (i.e., channels in their OFF states) at that instant as shown in Fig. 4. We assume that the pool of idle channels is treated as one *logical channel*, whose capacity is equal to $m \cdot C$ bandwidth-units. This can be realized by the Orthogonal Frequency Division Multiplexing (OFDM) techniques with adaptive and selective allocation of OFDM subcarriers, like Noncontiguous OFDM (NC-OFDM) proposed in [19].

4. In this paper, we consider stochastic PU patterns to address nondeterministic PU activities. There exist, however, some examples where PUs' usage patterns are predictable, e.g., TV transmitters operate according to their advertised broadcasting schedules. The deterministic case is left as our future work.

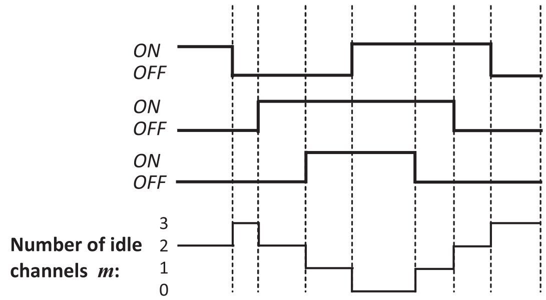


Fig. 4. Time-varying channel capacity.

3.2 Spectrum Auction Model

We consider a multiwinner periodic spectrum auction [20], [21] where an SB (auctioneer) auctions off the licensed channels periodically (e.g., hourly, daily, or even weekly) every $T_{auction}$, and multiple WSPs bid for the number of channels they want to lease. Once a WSP wins M channels, it pays the (leasing) price of $p_{bid}(M)$ *per unit-time* to obtain temporary rights to reuse the channels for the period of $T_{auction}$. After $T_{auction}$, the leased channels are returned to the licensees.

We make the following assumption on the bidding cost function $p_{bid}(M)$, $p_{bid}(0) = 0$, which is commonly accepted in the CR auction market literature [8], [22].

Assumption 1. *$p_{bid}(M)$ is a positive, nondecreasing and convex function of M.*

This assumption is reasonable because the winning bid is likely to increase faster than proportionally to M due to the competition between WSPs contending for the limited amount of spectrum resources auctioned off in the market. The actual form of $p_{bid}(M)$ should depend on the auction market, and hence, we assume $p_{bid}(M)$ is given a priori in order to focus on user-control issues. For an illustrative purpose, our simulation in Section 7 will use $p_{bid}(M) = D_1 \cdot M^{D_2}, D_2 \geq 1$, which was introduced in [8], [22] and satisfies Assumption 1. Note that D_2 represents the degree of competition in the auction.

3.3 Multiclass User QoS Model

A customer at a CR hotspot is a CR-capable device that is assumed to have a spectrum demand in one of the following K QoS-classes:

$$\mathbf{B} = (B_1, B_2, \ldots, B_k)^{\mathrm{T}},$$

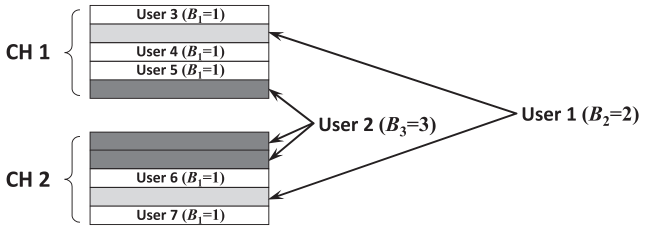


Fig. 5. An example of channel allocation with $M = 2$, $K = 3$, and $C = 5$.

where B_k is the bandwidth requirement of class-k customers and T represents "transpose." Note that we reserve k as the index of user class.

Based on selective OFDM subcarrier allocation, each CR user is assumed to be capable of tuning its antenna to any portion of the logical channel for its bandwidth assignment of B_k, as illustrated in Fig. 5. In this way, at a channel's OFF $\rightarrow$ ON transition, the users on the channel can be redistributed to other idle channels, by updating the mapping of OFDM subcarriers to the users. MAC-layer beaconing might be used to perform this remapping in real time.

We assume the arrival of class-k customers follows the Poisson distribution with rate λ_k, since the service requests from the CR users are made at the connection level and user-oriented connection requests are modeled well as a Poisson process [23]. For mathematical tractability, the service time of an in-service class-k customer is assumed exponentially distributed with mean $1/\mu_k$, capturing the reality of some applications such as phone-call traffic with exponentially distributed talk spurt [24].

3.4 End-User Pricing Model

The revenue of a WSP is generated by the CR end-users who pay fees for their opportunistic spectrum usage. The vector of the usage fees for K QoS-classes, in terms of price *per* unit-time *per* unit-bandwidth, is denoted by

$$\mathbf{p} = (p_1, p_2, \dots, p_K)^{\mathrm{T}}.$$

The arrival rate is price dependent, and therefore, it is represented by $\lambda_k(p_k)$. $\lambda_k(p_k)$ is a nonincreasing function of p_k, since a WSP advertising higher prices should expect less customer arrivals than the others offering lower prices. The actual price-arrival relationship depends strongly on the WSP's tariff and the degree of competition, which must be treated as a separate marketing problem. Here, we assume $\lambda_k(p_k)$ is given a priori and confine our discussion to the optimal user control. We leave $\lambda_k(p_k)$ as a general parameter in the analysis, and in Section 7 we will use an example of $\lambda_k(p_k) = \lambda_k^{max} e^{-\delta_k p_k}$ as introduced in [25], for an illustrative purpose. Here, λ_k^{max} indicates that the maximum user population at a CR hotspot is bounded, and δ_k represents the rate of decrease of the arrival rate as p_k increases, which is related to the degree of competition between WSPs.

4 SMDP Formulation

The profit maximization at a CR hotspot poses a unique challenge due to time-varying channel availability that necessitates joint user admission and eviction control by the CR WSP. In such a case, SMDP is a useful tool to determine the optimal actions achieving maximal profit, and thus in this section we formulate the system as an SMDP. We first show the validity of SMDP formulation for the problem considered, and then we derive the basic and essential components of the SMDP by accounting for time-varying channel availability and possible actions to be taken for user admission and eviction. In Section 5, we will use the derived components to construct our proposed LP algorithm that determines the optimal actions and achieves the maximal profit.

4.1 System State and State Space

We start with the definition of system state as

$$\mathbf{s} = (\mathbf{n}, \mathbf{w}), \quad \text{and} \begin{cases} \mathbf{n} = (n_1, n_2, \dots, n_k)^{\mathrm{T}}, \\ \mathbf{w} = (w_1, w_2, \dots, w_M)^{\mathrm{T}}, \end{cases} \tag{1}$$

where n_k is the number of class-k customers in service, and w_i is the channel state defined as

$$w_i = \begin{cases} 1, & \text{channel } i \text{ is occupied by PUs (i.e., ON)}, \\ 0, & \text{channel } i \text{ is } \textit{not} \text{ occupied by PUs (i.e., OFF)}. \end{cases}$$

Then, the sub-state-space of $\mathbf{w}$, denoted by $\Lambda_{\mathbf{w}}$, and the sub-state-space of $\mathbf{n}$ given $\mathbf{w}$, denoted by $\Lambda_{\mathbf{n}|\mathbf{w}}$, are defined as

$$\begin{aligned} \Lambda_{\mathbf{w}} &= \{\mathbf{w} : w_i \in \{0, 1\}\}, \\ \Lambda_{\mathbf{n}|\mathbf{w}} &= \{\mathbf{n} : n_k \geq 0,\ \mathbf{n}^{\mathrm{T}}\mathbf{B} \leq mC,\ m = (\mathbf{1} - \mathbf{w})^{\mathrm{T}}\mathbf{1}\}, \end{aligned}$$

where m is the number of idle channels and $\mathbf{1}$ is a vector of 1's. Therefore, the state space Λ is given as

$$\Lambda = \{\mathbf{s} : \mathbf{w} \in \Lambda_{\mathbf{w}}, \mathbf{n} \in \Lambda_{\mathbf{n}|\mathbf{w}}\}.$$

4.2 Possible Actions and Action Space

In our SMDP formulation, an *action* is taken and updated at each *decision epoch* under the chosen policy, where a natural choice of the decision epoch is the instant when a channel's state changes, i.e., 1) at a class-k customer's arrival/departure and 2) at channel i's state-transition (ON $\rightarrow$ OFF or OFF $\rightarrow$ ON).

We define the action at a certain decision epoch as

$$\boldsymbol{\alpha} = (\mathbf{a}, \mathbf{b}), \quad \text{and} \begin{cases} \mathbf{a} = (a_1, a_2, \dots, a_K)^{\mathrm{T}}, \\ \mathbf{b} = (b_1, b_2, \dots, b_K)^{\mathrm{T}}, \end{cases}$$

where $\mathbf{a}$ is the admission policy for future customer arrivals such that

$$a_k = \begin{cases} 0, & \textit{reject} \text{ all future class-}k \text{ arrivals}, \\ 1, & \textit{admit} \text{ all future class-}k \text{ arrivals}, \end{cases}$$

and $\mathbf{b}$ is the eviction policy for *in-service* customers where b_k indicates the number of class-k customers to be evicted *at the time of channel vacation*, i.e., at state-transition OFF $\rightarrow$ ON on a certain channel. That is, even if $\mathbf{b} \neq \mathbf{0}$, we do not evict any customer if the next event is an arrival, departure, or state-transition ON $\rightarrow$ OFF.

SMDP derives the optimal action α for each possible state. In other words, it determines state-dependent optimal actions $\alpha(\mathbf{s})$ for every state s.[5] Therefore, we define the action space $A(\mathbf{s})$ for a given state s as follows, from which the optimal action is determined

$$A(\mathbf{s}) = \begin{cases} \{\boldsymbol{\alpha} : \mathbf{a} = \mathbf{0},\ \mathbf{b} = \mathbf{0}\}, & \text{if } m = 0, \\ \{\boldsymbol{\alpha} : \mathbf{a} \in A_1(\mathbf{s}),\ \mathbf{b} \in A_2(\mathbf{s})\}, & \text{otherwise,} \end{cases} \tag{2}$$

with $A_1(\mathbf{s})$ and $A_2(\mathbf{s})$ being defined as

$$\begin{aligned} A_1(\mathbf{s}) = \{\mathbf{a} : a_k \in \{0,1\};& \\ a_k = 0 \text{ if } (\mathbf{n} + \mathbf{u}_k)^{\mathrm{T}}\mathbf{B} > mC\},& \end{aligned} \tag{3}$$

$$\begin{aligned} A_2(\mathbf{s}) = \{\mathbf{b} :\ & 0 \le b_k \le n_k; \\ & \mathbf{b} = \mathbf{0} \text{ if } \mathbf{n}^{\mathrm{T}}\mathbf{B} \le (m-1)C; \\ & (\mathbf{n} - \mathbf{b} + \mathbf{u}_k)^{\mathrm{T}}\mathbf{B} > (m-1)C, \text{ for } \forall k \text{ s.t.} \\ & b_k \ne 0, \text{ and } (\mathbf{n} - \mathbf{b})^{\mathrm{T}}\mathbf{B} \le (m-1)C\}, \end{aligned} \tag{4}$$

where $\mathbf{u}_j$ is a unit vector with a single 1 at the jth position and 0's elsewhere. In the definition of $A_2(\mathbf{s})$, the constraint

$$(\mathbf{n} - \mathbf{b} + \mathbf{u}_k)^{\mathrm{T}}\mathbf{B} > (m-1)C \text{ for } \forall k \text{ s.t. } b_k \ne 0,$$

implies that b should be *minimal* so as not to evict more in-service customers than necessary. That is, b is *not* minimal if there exists k such that $\mathbf{b}' = \mathbf{b} - \mathbf{u}_k$ and $(\mathbf{n} - \mathbf{b}')^{\mathrm{T}}\mathbf{B} \le (m-1)C$, i.e., evicting one less customer than b still fits in $(m-1)$ channels. Obviously, the choice of minimal b for given s is not unique. For example, in case $m = 2$, $C = 5$, and $\mathbf{B} = (1, 2)$, there are three possible minimal b's for $\mathbf{n} = (5, 2)$: $\mathbf{b} = (4, 0)$, $\mathbf{b} = (2, 1)$ and $\mathbf{b} = (0, 2)$.

4.3 Validity of SMDP Formulation

Before deriving the other essential components of the SMDP, we need to check if the system under consideration can be modeled as an SMDP. In [26], an SMDP is defined as follows:

Definition 1. *A dynamic system is said to be a semi-Markov decision process if the following property is satisfied: if at a decision epoch the action $\boldsymbol{\alpha}$ is chosen in state $\mathbf{s}$, then the time until the state at, and revenue/cost incurred until, the next decision epoch depend only on $\mathbf{s}$ and $\boldsymbol{\alpha}$.*

To check if a system under consideration satisfies Definition 1, we first consider 1) a combined process of K-class user arrivals and departures, 2) M independent processes of channel-state transitions, and 3) a combined process of 1 and 2.

4.3.1 A Combined Process of User Arrivals and Departures

Interarrival and interdeparture times of class-k customers are all assumed to be exponentially distributed. Since the arrival and departure processes are independent of each other, the interevent time, denoted by T_0, of the combined random process is also exponentially distributed with mean $1/\rho$, where $\rho := \mathbf{n}^{\mathrm{T}}\boldsymbol{\mu} + \mathbf{a}^{\mathrm{T}}\boldsymbol{\lambda}$, $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_K)^{\mathrm{T}}$, and $\boldsymbol{\mu} = (\mu_1, \mu_2, \ldots, \mu_K)^{\mathrm{T}}$.

4.3.2 M Independent Processes of Channel-State Transitions

In [1], we considered exponentially distributed ON and OFF periods with which the time until the next channel-state transition depends only on the current channel states w due to the memoryless property, and proved that the overall process indeed satisfies the conditions in Definition 1. In this paper, we will extend the preliminary result in [1] to the case of generally distributed ON/OFF periods.

For generally distributed ON/OFF periods, however, the time until the next state-transition on channel i depends not only on w_i but also on the elapsed time since the last state-transition, denoted by e_i. To satisfy the conditions in Definition 1, the previous definition of state s can be extended as

$$\mathbf{s} = (\mathbf{n}, \mathbf{w}, \mathbf{e}), \quad \mathbf{e} = (e_1, e_2, \ldots, e_M)^{\mathrm{T}}.$$

This approach, however, is impractical since e_i is continuous, introducing infinitely many possible states.

To derive a mathematically tractable but reasonably accurate model, we take an approximation-based approach using renewal theory [14] as follows:

Proposition 1 (Cox [14]). *Given that a renewal process of channel i has started a long time ago, the residual time in the current state, regardless of how much time has elapsed since the last state-transition, has the pdf of*

$$\begin{aligned} \big(1 - F_{T^i_{OFF}}(t)\big)/E\big[T^i_{OFF}\big], t > 0, \quad \text{if } w_i = 0, \\ \big(1 - F_{T^i_{ON}}(t)\big)/E\big[T^i_{ON}\big], t > 0, \quad \text{if } w_i = 1, \end{aligned} \tag{5}$$

where $F_{T^i_{OFF}}(t)$ and $F_{T^i_{ON}}(t)$ are the cumulative distribution functions (cdfs) of T^i_{OFF} and T^i_{ON}.

Proposition 1 indicates that as the current time progresses farther away from the time origin of a renewal channel, the pdf of the remaining time until the next state-transition will converge to (5). Note that exponential distribution is the only instance with which (5) coincides with $f_{T^i_{OFF}}(t)$ and $f_{T^i_{ON}}(t)$.

Using this property, we can keep the definition of s same as (1) while approximating the system as an SMDP because the pdfs in (5) do not depend on e_i's and are thus memoryless. In Section 7, we will show that this approximation, in fact, produces reasonably accurate results. It should be noted, however, that such approximation yields a suboptimal performance even though the user control is optimally derived from the given approximate model.

4.3.3 The Overall Combined Process

Using the fact that the combined arrival/departure process and renewal processes of M channels are independent of each other, the cdf of the remaining time T until the *next* decision epoch is given as

5. Since $\boldsymbol{\alpha}$ is state-dependent, the admission and eviction policy will be updated every time the system state s changes due to arrivals, departures, and channel state transitions. For example, when the current admission policy indicates rejection of all class-k arrivals, it only holds as long as the system stays at the same state.

$$\begin{aligned} P(T \le t) &= 1 - P(T = \min(T_0, T_1, \ldots, T_M) > t) \\ &= 1 - P(T_0 > t, T_1 > t, \ldots, T_M > t) \\ &= 1 - \prod_{l=0}^{M} P(T_l > t) = 1 - \prod_{l=0}^{M} (1 - P(T_l \le t)), \end{aligned} \tag{6}$$

where T_l $(1 \le l \le M)$ is a random variable representing the residual time until the next state-transition on channel l whose pdf is given as (5).

As a result, the system model considered in this paper becomes an SMDP since it possesses the properties in Definition 1:

- The time until the next decision epoch depends only on $(\mathbf{s}, \boldsymbol{\alpha})$ since (6) is a function of $\mathbf{n}$, $\mathbf{a}$, $\mathbf{w}$.
- The state $\mathbf{s}' = (\mathbf{n}', \mathbf{w}')$ at the next decision epoch depends only on $\mathbf{s}$ and $\boldsymbol{\alpha}$ such that
  - $\mathbf{n}' = \mathbf{n} + \mathbf{u}_k$, $\mathbf{w}' = \mathbf{w}$, at a class-k user's arrival,
  - $\mathbf{n}' = \mathbf{n} - \mathbf{u}_k$, $\mathbf{w}' = \mathbf{w}$, at a class-k user's departure,
  - $\mathbf{n}' = \mathbf{n} - \mathbf{b}$, $\mathbf{w}' = \mathbf{w} + \mathbf{u}_i$, at channel i's OFF $\rightarrow$ ON transition, and
  - $\mathbf{n}' = \mathbf{n}$, $\mathbf{w}' = \mathbf{w} - \mathbf{u}_i$, at channel i's ON $\rightarrow$ OFF transition.
- The revenue and cost accrued until the next decision epoch depend only on $\mathbf{s}$ and $\boldsymbol{\alpha}$ since they are a function of $\mathbf{n}$, $\mathbf{b}$, and the time until the next decision epoch. The definition of revenue and cost will be detailed in Section 4.6.

4.4 Decision Epochs

The expected time between two decision epochs, denoted by $\tau_{\mathbf{s}}(\boldsymbol{\alpha})$, is determined as

$$\tau_{\mathbf{s}}(\boldsymbol{\alpha}) = \int_0^{\infty} t f_T(t) dt, \; f_T(t) = \frac{dP(T \le t)}{dt},$$

where $P(T \le t)$ is from (6).

For example, with exponentially distributed ON and OFF durations with mean $1/\mu_{ON}^i$ and $1/\mu_{OFF}^i$, the expected time between two decision epochs is determined as

$$\tau_{\mathbf{s}}(\boldsymbol{\alpha}) = \left[\mathbf{n}^{\mathrm{T}}\boldsymbol{\mu} + \mathbf{a}^{\mathrm{T}}\boldsymbol{\lambda} + (\mathbf{1} - \mathbf{w})^{\mathrm{T}}\boldsymbol{\mu_{OFF}} + \mathbf{w}^{\mathrm{T}}\boldsymbol{\mu_{ON}}\right]^{-1},$$

where $\boldsymbol{\mu_{OFF}} = (\mu_{OFF}^1, \mu_{OFF}^2, \ldots, \mu_{OFF}^M)^{\mathrm{T}}$ and $\boldsymbol{\mu_{ON}} = (\mu_{ON}^1, \mu_{ON}^2, \ldots, \mu_{ON}^M)^{\mathrm{T}}$. This result can be understood intuitively as follows: Since channel i's residual time in ON (or OFF) state is exponentially distributed with mean $1/\mu_{ON}^i$ (or $1/\mu_{OFF}^i$), the time between state-transitions becomes exponentially distributed with mean $1/((1 - w_i)\mu_{OFF}^i + w_i\mu_{ON}^i)$. Therefore, with M independent channels, the combined random process is also exponentially distributed with mean $[(\mathbf{1} - \mathbf{w})^{\mathrm{T}}\boldsymbol{\mu_{OFF}} + \mathbf{w}^{\mathrm{T}}\boldsymbol{\mu_{ON}}]^{-1}$. As a result, the combined process of user arrivals/departures with M channel state-transitions becomes exponentially distributed with mean $[\mathbf{n}^{\mathrm{T}}\boldsymbol{\mu} + \mathbf{a}^{\mathrm{T}}\boldsymbol{\lambda} + (\mathbf{1} - \mathbf{w})^{\mathrm{T}}\boldsymbol{\mu_{OFF}} + \mathbf{w}^{\mathrm{T}}\boldsymbol{\mu_{ON}}]^{-1}$.

4.5 State-Transition Probability

The probability that the state of the SMDP switches from $\mathbf{s} = (\mathbf{n}, \mathbf{w})$ to $\mathbf{s}' = (\mathbf{n}', \mathbf{w}')$ at the next decision epoch is given as in (7), where T_a, T_{-a}, T_d, and T_{-d} are exponentially distributed random variables with mean $1/a_k\lambda_k$, $1/(\rho - a_k\lambda_k)$, $1/n_k\mu_k$, and $1/(\rho - n_k\mu_k)$.

$$\begin{aligned} p_{\mathbf{s},\mathbf{s}'}(\boldsymbol{\alpha}) &= \begin{cases} P(T_a \le T_{-a}, T_a \le T_1, \ldots, T_a \le T_M), \\ \quad \text{at a class-}k\text{ user's arrival}, \\ P(T_d \le T_{-d}, T_d \le T_1, \ldots, T_d \le T_M), \\ \quad \text{at a class-}k\text{ user's departure}, \\ P(T_i \le T_0, T_i \le T_1, \ldots, T_i \le T_M), \\ \quad \text{at channel } i\text{'s OFF} \leftrightarrow \text{ON}, \end{cases} \\ &= \begin{cases} \int_0^{\infty}\int_{t_a}^{\infty}\cdots\int_{t_a}^{\infty}\{f_{T_a}(t_a)f_{T_{-a}}(t_{-a})f_{T_1}(t_1) \\ \quad \cdots f_{T_M}(t_M)\}dt_{-a}dt_1\cdots dt_M dt_a, \\ \quad \text{at a class-}k\text{ user's arrival}, \\ \int_0^{\infty}\int_{t_d}^{\infty}\cdots\int_{t_d}^{\infty}\{f_{T_d}(t_d)f_{T_{-d}}(t_{-d})f_{T_1}(t_1) \\ \quad \cdots f_{T_M}(t_M)\}dt_{-d}dt_1\cdots dt_M dt_d, \\ \quad \text{at a class-}k\text{ user's departure}, \\ \int_0^{\infty}\int_{t_i}^{\infty}\cdots\int_{t_i}^{\infty}\{f_{T_0}(t_0)f_{T_1}(t_1) \\ \quad \cdots f_{T_M}(t_M)\}dt_0 \underbrace{dt_1\cdots dt_M}_{\text{except } t_i} dt_i, \\ \quad \text{at channel } i\text{'s OFF} \leftrightarrow \text{ON}. \end{cases} \end{aligned} \tag{7}$$

For example, for exponentially distributed ON/OFF periods, we have

$$p_{\mathbf{s},\mathbf{s}'}(\boldsymbol{\alpha}) = \begin{cases} a_k\lambda_k\tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n} + \mathbf{u}_k, \; \mathbf{w}' = \mathbf{w}, \\ & (\text{class } k \text{ arrival}), \\ n_k\mu_k\tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n} - \mathbf{u}_k, \; \mathbf{w}' = \mathbf{w}, \\ & (\text{class } k \text{ departure}), \\ \mu_{OFF}^i\tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n} - \mathbf{b}, \; \mathbf{w}' = \mathbf{w} + \mathbf{u}_i, \\ & (\text{channel } i\text{: OFF} \rightarrow \text{ON}), \\ \mu_{ON}^i\tau_{\mathbf{s}}(\boldsymbol{\alpha}), & \mathbf{n}' = \mathbf{n}, \; \mathbf{w}' = \mathbf{w} - \mathbf{u}_i, \\ & (\text{channel } i\text{: ON} \rightarrow \text{OFF}). \end{cases}$$

4.6 Revenue and Reimbursement Cost

Let $r_{\mathbf{s}}(\boldsymbol{\alpha})$ and $c_{\mathbf{s}}(\boldsymbol{\alpha})$ denote the expected revenue and the cost incurred by customers until the next decision epoch if action $\boldsymbol{\alpha}$ is chosen at state $\mathbf{s}$, respectively. Since the revenue comes from the usage fee paid by the admitted customers, $r_{\mathbf{s}}(\boldsymbol{\alpha})$ is given as

$$r_{\mathbf{s}}(\boldsymbol{\alpha}) = \sum_k p_k B_k n_k \tau_{\mathbf{s}}(\boldsymbol{\alpha}),$$

Assuming a fixed amount of reimbursement I_k for an evicted class-k customer,[6] $c_{\mathbf{s}}(\boldsymbol{\alpha})$ is

$$c_{\mathbf{s}}(\boldsymbol{\alpha}) = \sum_k I_k b_k \cdot q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha}),$$

where $\tau_{\mathbf{s}}(\boldsymbol{\alpha})$ does not contribute to the equation since the reimbursement is a one-time cost at channel vacation. $q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha})$ is the probability that the event of channel vacation will happen at the next decision epoch. We also let $q_{\mathbf{s}}^{A,k}(\boldsymbol{\alpha})$ denote the probability that the event of a class-k user's arrival will occur and then be accepted at the next decision epoch. Then, we have

6. We consider fixed reimbursement as an exemplary case of many possible reimbursement policies. It should be noted that other types of I_k can also be applied (e.g., reimbursement proportional to the usage charge) by only updating the form of $c_{\mathbf{s}}(\boldsymbol{\alpha})$ without changing the current formulation of the problem.

$$q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha}) = \sum_{\substack{\mathbf{s}'\in\Lambda \text{ s.t. } \mathbf{n}'=\mathbf{n}-\mathbf{b},\\ \mathbf{w}'=\mathbf{w}+\mathbf{u}_i \text{ for some } i}} p_{\mathbf{s},\mathbf{s}'}(\boldsymbol{\alpha}),$$

$$q_{\mathbf{s}}^{\mathcal{A},k}(\boldsymbol{\alpha}) = \sum_{\substack{\mathbf{s}'\in\Lambda \text{ s.t.}\\ \mathbf{n}'=\mathbf{n}+\mathbf{u}_k,\mathbf{w}'=\mathbf{w}}} p_{\mathbf{s},\mathbf{s}'}(\boldsymbol{\alpha}).$$

5 Optimal User Control via a Linear Programming Algorithm

In this section, we propose an LP algorithm based on the essential SMDP components derived in the previous section, that can maximize the profit at CR hotspots by determining the optimal action $\boldsymbol{\alpha}$ at each possible system state s subject to the QoS constraints such as keeping the blocking and dropping probabilities below certain thresholds. There are three well-known methods for optimally solving the SMDP problem: *policy-iteration*, *value-iteration*, and *linear programming* [26], and of these, we adopt LP because it is the best-known to model the QoS-constrained optimization problem thanks to its flexibility to include additional equality and inequality constraints.

5.1 Linear Programming SMDP Algorithm: Constrained QoS

Here, we formulate a 3-step LP algorithm with the constraints on the probability of blocking class-k arrivals (denoted by P_b^k), and the probability of dropping/evicting class-k in-service customers (denoted by P_d^k). This 3-step algorithm follows the general format of the LP algorithm recommended in [26], but its mathematical content is our own development.

***Step 1*:** Find the optimal basic solution $z^*_{\mathbf{s},\boldsymbol{\alpha}}$ to the following linear programming problem ($z_{\mathbf{s},\boldsymbol{\alpha}} \geq 0$):

$$\textbf{Maximize} \sum_{\mathbf{s}\in\Lambda}\sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} (r_{\mathbf{s}}(\boldsymbol{\alpha}) - c_{\mathbf{s}}(\boldsymbol{\alpha}))z_{\mathbf{s},\boldsymbol{\alpha}}$$

Subject to

$$\begin{aligned}
&\sum_{\boldsymbol{\alpha}\in A(\mathbf{s}')} z_{\mathbf{s}',\boldsymbol{\alpha}} - \sum_{\mathbf{s}\in\Lambda}\sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} p_{\mathbf{s},\mathbf{s}'}(\boldsymbol{\alpha})z_{\mathbf{s},\boldsymbol{\alpha}} = 0,\ \mathbf{s}'\in\Lambda,\\
&\sum_{\mathbf{s}\in\Lambda}\sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} \tau_{\mathbf{s}}(\boldsymbol{\alpha})z_{\mathbf{s},\boldsymbol{\alpha}} = 1,\\
&P_b^k \leq \gamma^k_{block},\ \forall k,\\
&P_d^k \leq \gamma^k_{drop},\ \forall k,
\end{aligned} \tag{8}$$

where P_b^k and P_d^k in (8) are determined as

$$\begin{aligned}
P_b^k &= \sum_{\mathbf{s}\in\Lambda}\sum_{\substack{\boldsymbol{\alpha}\in A(\mathbf{s})\\ \text{s.t. } a_k=0}} \tau_{\mathbf{s}}(\boldsymbol{\alpha})z_{\mathbf{s},\boldsymbol{\alpha}},\\
P_d^k &= \frac{\mathcal{V}^k}{\mathcal{A}^k},
\end{aligned} \tag{9}$$

where

$$\mathcal{A}^k = \sum_{\mathbf{s}\in\Lambda}\sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} \tau_{\mathbf{s}}(\boldsymbol{\alpha})z_{\mathbf{s},\boldsymbol{\alpha}}q_{\mathbf{s}}^{\mathcal{A},k}(\boldsymbol{\alpha})\cdot\frac{1}{\tau_{\mathbf{s}}(\boldsymbol{\alpha})},$$

$$\mathcal{V}^k = \sum_{\mathbf{s}\in\Lambda}\sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} \tau_{\mathbf{s}}(\boldsymbol{\alpha})z_{\mathbf{s},\boldsymbol{\alpha}}q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha})\cdot\frac{b_k}{\tau_{\mathbf{s}}(\boldsymbol{\alpha})},$$

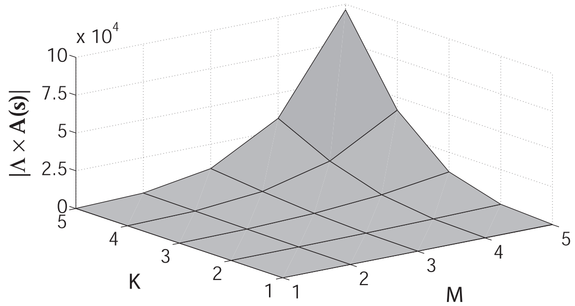


Fig. 6. The size of the search space according to (M, K).

and $z_{\mathbf{s},\boldsymbol{\alpha}} = x_{\mathbf{s},\boldsymbol{\alpha}}/\tau_{\mathbf{s}}(\boldsymbol{\alpha})$ with $x_{\mathbf{s},\boldsymbol{\alpha}}$ denoting the fraction of time that the system is in state s when action $\boldsymbol{\alpha}$ is chosen. Therefore, $\mathcal{A}^k$ implies the expected number of class-k accepted arrivals per unit-time and $\mathcal{V}^k$ implies the expected number of class-k evictions per unit-time.

The form of P_d^k makes this a nonlinear programming (NLP) problem. Fortunately, however, by properly manipulating the constraint $P_d^k \leq \gamma^k_{drop}$, it can be converted to a LP problem as follows:

$$\begin{aligned}
&\frac{\mathcal{V}^k}{\mathcal{A}^k} \leq \gamma^k_{drop}\\
&\quad\Rightarrow \mathcal{V}^k - \gamma^k_{drop}\mathcal{A}^k \leq 0\\
&\quad\Rightarrow \sum_{\mathbf{s}\in\Lambda}\sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} \left(q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha})\cdot b_k - \gamma^k_{drop}q_{\mathbf{s}}^{\mathcal{A},k}(\boldsymbol{\alpha})\cdot 1\right)z_{\mathbf{s},\boldsymbol{\alpha}} \leq 0,
\end{aligned}$$

which is a linear constraint on $z_{\mathbf{s},\boldsymbol{\alpha}}$'s.

***Step 2*:** Start with a nonempty set

$$S := \left\{\mathbf{s}\middle|\ \sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} z^*_{\mathbf{s},\boldsymbol{\alpha}} > 0\right\},$$

and for any state $\mathbf{s}\in S$, set the decision as

$$R^*(\mathbf{s}) := \boldsymbol{\alpha} \text{ for some } \boldsymbol{\alpha} \text{ such that } z^*_{\mathbf{s},\boldsymbol{\alpha}} > 0.$$

***Step 3*:** If $S = \Lambda$, then the algorithm terminates with the optimal policy R^*. Otherwise, determine some state $\mathbf{s}\notin S$ and action $\boldsymbol{\alpha}\in A(\mathbf{s})$ such that $p_{\mathbf{s},\mathbf{s}'}(\boldsymbol{\alpha}) > 0$ for some $\mathbf{s}'\in S$. For the chosen s, set $R^*(\mathbf{s}) := \boldsymbol{\alpha}$ and update $S := S\cup\{\mathbf{s}\}$, and then repeat Step 3.

By repeatedly performing Step 3, the algorithm runs until S becomes Λ. The computational complexity of executing this final step is trivial since $p_{\mathbf{s},\mathbf{s}'}(\boldsymbol{\alpha}) > 0$ has already been computed in Step 1 for all possible combinations of $(\mathbf{s},\mathbf{s}',\boldsymbol{\alpha})$. In addition, the optimality of the derived policy is guaranteed in [26], although the algorithm may not produce a unique solution due to the conditions "some $\boldsymbol{\alpha}$" and "some s′" in Steps 2 and 3.

Then, the *optimal profit* g^* per unit-time is determined as

$$g^* = \sum_{\substack{\mathbf{s}\in\Lambda\\ \boldsymbol{\alpha}=R^*(\mathbf{s})}} (r_{\mathbf{s}}(\boldsymbol{\alpha}) - c_{\mathbf{s}}(\boldsymbol{\alpha}))z_{\mathbf{s},\boldsymbol{\alpha}} - p_{bid}(M). \tag{10}$$

5.2 Complexity of SMDP Algorithm

The complexity of the proposed algorithm is measured by the size of the search space $\Lambda\times A(\mathbf{s})$. In Fig. 6, one can see that $|\Lambda\times A(\mathbf{s})|$ increases almost exponentially as M or K increases, because $|\Lambda_{\mathbf{w}}| = 2^M$ and $|A_1(\mathbf{s})| = 2^K$ in the worst case.

However, the complexity issue can be managed properly in the real scenarios due to the following two reasons.

First, in commercial applications, a reasonable range of (M, K) could be $1 \le K, M \le 3$ in which case we have a moderate and reasonable level of complexity since $|\Lambda \times A(\mathbf{s})| \le 2{,}167$ in Fig. 6. For example, $K = 3$ for a service with Gold, Silver, and Bronze classes, and $K = 2$ for a service with Premium and Basic classes. In fact, the premium/basic classification is commonly found in today's commercial Wi-Fi hotspot services. In addition, if we compare the *effective* capacity of our Wi-Fi-like service over whitespaces with that of the traditional Wi-Fi, a CR hotspot utilizing M ON/OFF channels has $M \cdot (1-u)C$ provided the channels have a similar utilization factor of u such that

$$u = E\big[T^i_{ON}\big] / \big\{E\big[T^i_{ON}\big] + E\big[T^i_{OFF}\big]\big\},$$

while the traditional Wi-Fi utilizing a single and always-idle channel has C. In practice, licensed bands with lower utilization (e.g., $u < 0.5$) have more whitespaces, and thus they are more preferred for DSA deployment. In such channels, it is sufficient to have $M \ge 2$ to achieve a capacity equal to or larger than C. As a result, most practical scenarios will incur manageable complexity.

Next, the LP can be solved *offline* before a WSP starts a new spectrum-leasing period (i.e., $T_{auction}$), so the derived optimal control can be stored in a database. Using the database, the WSP can perform user admission control in real time by simply looking up the database upon every user arrival/departure or channel ON/OFF transition.

6 Prioritized Multiclass User Control

So far, we have investigated the optimal user control at CR hotspots for multiclass customers, each requiring a different level of QoS, without assuming any priority in service provisioning between them. In this section, we introduce the case when different priority is given to each user class, and discuss how optimal actions are derived in such a case.

Without loss of generality, we assume that a larger-index class is given higher priority, i.e., class K gets the highest priority and class 1 gets the lowest priority in either admission or eviction control. This type of service may be viewed as a hybrid service of two types of today's commercial Wi-Fi services—*free* (e.g., at some coffee shops) and *charged* Wi-Fi access (e.g., AT&T Wi-Fi access). For example, for $K = 2$, by making $p_2 > 0$ and $p_1 = 0$, customers with less important jobs may choose a best-effort free service (class 1) while customers with important or resource demanding jobs, such as multimedia applications, may choose a reliable-but-fee-paying service (class 2).

In prioritized admission control (p-AC), user eviction is allowed not only at the time of channel vacation but also at the customer arrivals. Under this new policy, an arriving user is *always* accepted for the service *unless* there is no room even after evicting all lower class users. When accepting the newly arrived user, the system should evict a certain number of lower priority in-service users, but no more than necessary. To model such an action, we need to modify the action space $A_1(\mathbf{s})$ in (3) to express the prioritized admission control, as shown in (11).

$$A_1(\mathbf{s}) = \left\{ \mathbf{a} : a_K = \begin{cases} 1, & \text{if } (n_K+1)B_K \le mC, \\ 0, & \text{otherwise,} \end{cases} \quad \text{and} \right.$$
$$\left. a_{k<K} = \begin{cases} 1, & \text{if } \sum_{l=k+1}^{K} n_l B_l + (n_k+1)B_k \le mC, \\ 0, & \text{otherwise.} \end{cases} \right\}. \tag{11}$$

In prioritized eviction control (p-EC), an in-service user is *never* evicted from the system *unless* there is no room even after evicting all lower class users. This rule is modeled by modifying the action space $A_2(\mathbf{s})$ in (4) as shown in (12).

$$\begin{aligned} A_2(\mathbf{s}) = \Bigg\{ \mathbf{b} : b_K = \beta_K, \text{ and} \\ b_{k<K} = \begin{cases} n_k, & \text{if } \sum_{l=k+1}^{K} n_l B_l \ge (m-1)C, \\ \beta_k, & \text{otherwise.} \end{cases} \Bigg\}, \\ \beta_K = \min\{n_K \ge \beta \ge 0 : (n_K - \beta)B_K \le (m-1)C\}, \\ \beta_k = \min\Bigg\{ n_k \ge \beta \ge 0 : \sum_{l=k+1}^{K} n_l B_l \\ + (n_k - \beta)B_k \le (m-1)C \Bigg\}. \end{aligned} \tag{12}$$

Note that if both p-AC and p-EC are employed, the action at given state s is uniquely determined as can be seen from (11) and (12). In case either p-AC or p-EC is not applied, however, the LP algorithm in Section 5 should still be used in deriving optimal actions.

For the prioritized service, we may need to modify/redefine some SMDP components introduced earlier. First, to differentiate user eviction at customer arrivals from user eviction at channel vacation, we redefine the action $\boldsymbol{\alpha}$ as

$$\boldsymbol{\alpha} = (\mathbf{a}, \mathbf{b}, \mathbf{c}^1, \ldots, \mathbf{c}^K). \tag{13}$$

In (13), $\mathbf{c}^\kappa$, $1 \le \kappa \le K$, is defined as

$$\mathbf{c}^\kappa = \left(c^\kappa_1, c^\kappa_2, \ldots, c^\kappa_k\right)^{\mathrm{T}},$$

where c^κ_k is the number of class-k in-service users to be evicted if the next event is a class-κ customer arrival, which is determined as in (14).

$$c^\kappa_k = \begin{cases} 0, & \text{if } \kappa \le k, \\ n_k, & \text{if } \kappa > k, \sum_{l=k+1}^{K} n_l B_l + B_\kappa > mC, \\ \min\Big\{ \omega : 0 \le \omega \le n_k, \text{ and } \sum_{l=k+1}^{K} n_l B_l + B_\kappa + (n_k - \omega)B_k \le mC \Big\}, & \text{otherwise.} \end{cases} \tag{14}$$

Then, upon a class-κ customer's arrival, we have $\mathbf{n}' = \mathbf{n} + \mathbf{u}_\kappa a_\kappa - \mathbf{c}^\kappa a_\kappa$ instead of $\mathbf{n}' = \mathbf{n} + \mathbf{u}_\kappa$. Note that when p-AC is not applied, we set $c^\kappa_k = 0, \forall (k, \kappa)$.

TABLE 1
The List of Common Test Parameters

Channel	$C = 5$, $E[T^i_{OFF}] = 10$, $E[T^i_{ON}] = 5$, $\forall i$
Auction	$p_{bid}(M) = 0.7 \cdot M^2$
Customers	$B_k = k, \forall k$, $\lambda_k(p_k) = \lambda_k^{max} e^{-p_k}$, $\mu_k = 1/7, \forall k$, *where* $(\lambda_1^{max}, \lambda_2^{max}, \lambda_3^{max}) = (5.5, 4.5, 5.5)$
Eviction	$I_k = \epsilon_I \times p_k B_k / \mu_k$, $\epsilon_I = 0.5$

TABLE 2
The List of Test-Specific Parameters

Section	M	K	γ_{block}	γ_{drop}	$\mathbf{p}$
7.1	*3*	*2*	*1.0*	*1.0*	$(1, 1.5)^{\mathrm{T}}$
7.2	*1-5*	*3*	*1.0*	*1.0*	$(1, 1.5, 2)^{\mathrm{T}}$
7.3.1	*1-5*	*3*	*1.0*	*1.0*	$(1, 1.5, 2)^{\mathrm{T}}$
7.3.2	*1-5*	*3*	*1.0*	*1.0*	$(1, 1.5, 2)^{\mathrm{T}}$
7.3.3	*3*	*2*	*1.0*	*1.0*	$0.5 \leq p_1, p_2 \leq 3.5$
7.4	*4*	*3*	*0.65-0.85*	*0.25-0.65*	$(1, 1.5, 2)^{\mathrm{T}}$

Next, P_d^k is redefined as

$$P_d^k = \frac{\mathcal{V}^k + \mathcal{V}_{arr}^k}{\mathcal{A}^k},$$

$$\mathcal{V}_{arr}^k = \sum_{\mathbf{s}\in\Lambda} \sum_{\boldsymbol{\alpha}\in A(\mathbf{s})} \sum_{\kappa} \tau_{\mathbf{s}}(\boldsymbol{\alpha}) z_{\mathbf{s},\boldsymbol{\alpha}} q_{\mathbf{s}}^{\mathcal{A},\kappa}(\boldsymbol{\alpha}) \cdot \frac{c_k^\kappa a_\kappa}{\tau_{\mathbf{s}}(\boldsymbol{\alpha})},$$

where $\mathcal{A}^k$ and $\mathcal{V}^k$ are the same as in Section 5, and $\mathcal{V}_{arr}^k$ implies the expected number of class-k evictions per unit-time due to the arrivals with higher priority.

Finally, the reimbursement cost $c_{\mathbf{s}}(\alpha)$ is updated as

$$c_{\mathbf{s}}(\boldsymbol{\alpha}) = \sum_k I_k b_k \cdot q_{\mathbf{s}}^{\mathcal{V}}(\boldsymbol{\alpha}) + \sum_\kappa \sum_k I_k c_k^\kappa a_\kappa \cdot q_{\mathbf{s}}^{\mathcal{A},\kappa}(\boldsymbol{\alpha}).$$

7 Performance Evaluation

In this section, we first present the system state of a WSP according to the optimal user admission/eviction control, and show the robustness of the approximation of (5) in Section 4.3. Then, we evaluate the impact of various system parameters on the achieved profit g^* such as the optimal control, the number of leased channels M, and the service tariff $\mathbf{p}$. Finally, we introduce the tradeoff between two QoS metrics, the probability of blocking and the probability of dropping.

In all simulation experiments, we randomly generated customer arrivals and departures according to the Poisson distributions as we assumed, and randomly produced ON/OFF periods according to either exponential or Erlang distribution.[7] At each decision epoch, an action is taken in accordance with the set of optimal actions determined by the analysis (via solving the LP). In addition, we consider the unprioritized multiclass service.

Each simulation ran for 3,000 time units and the same simulation repeats 10 times to observe its average performance. The simulation parameters used in this section are summarized in Tables 1 and 2. In Table 1, I_k is set to be $\epsilon_I \cdot 100\%$ of the average usage charge until the normal departure (not eviction) of an admitted class-k user. In Table 2, $\gamma_{block} = \gamma_{block}^k$ and $\gamma_{drop} = \gamma_{drop}^k$ for all k.

7.1 System State Transition by Optimal Control

Fig. 7 illustrates the optimal actions derived by the proposed SMDP algorithm in the form of state-transition diagram, when $M = 3$ and $K = 2$. For simplicity, the state-transition diagram is drawn for $m = 2$ (i.e., when there are two idle channels) and only the transitions by the user arrivals are presented. As shown, the optimal admission policy deliberately rejects certain arrivals to maximize the profit, disabling some possible state-transitions (shaded regions). It is also observed that the derived optimal control is not threshold-type [27] as found in traditional networks with static spectrum availability.

7.2 Approximation Accuracy of (5)

Fig. 8 shows the difference between the analytically predicted results given by (10) and the simulation results, for two different ON/OFF distributions: exponential and Erlang. To focus on the accuracy of our analysis, we plot $g^* + p_{bid}(M)$. In Fig. 8a, it is seen that the two results match well each other since there exists no approximation error in case ON/OFF periods are exponentially distributed. On the other hand, when ON/OFF periods are Erlang-distributed as in Fig. 8b, there exists only up to 4.4 percent difference between the simulated and analytic results showing that the approximation in (5) produces reasonably accurate results. The difference gets slightly larger as M grows due to the increase in the number of channels where approximation is used.

7.3 Achieved Optimal Profit by SMDP Algorithm

We now show the optimal profit achieved by the proposed SMDP under various test scenarios. We first compare the optimal profit of SMDP with the profit of the simple complete-sharing (CS) algorithm [10], and show the impact of the system parameters on the achieved profit such as the number of leased channels and the service tariff.

7.3.1 Optimal versus Nonoptimal Control

We compare the performance of the proposed SMDP algorithm with the CS algorithm in terms of g^* and

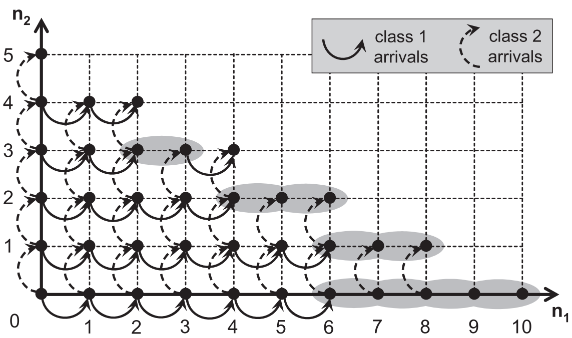
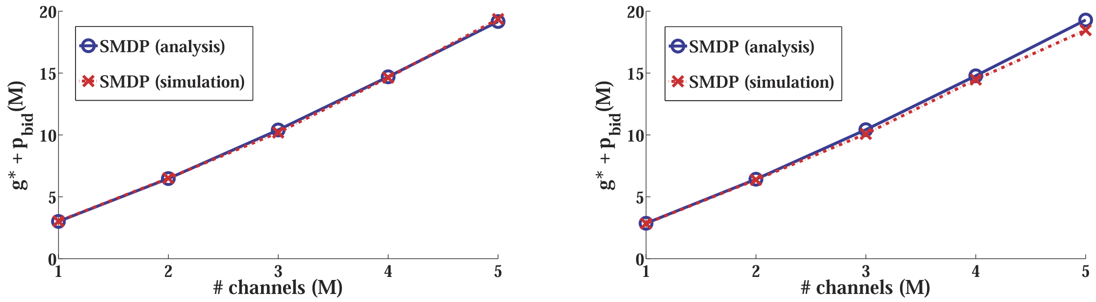


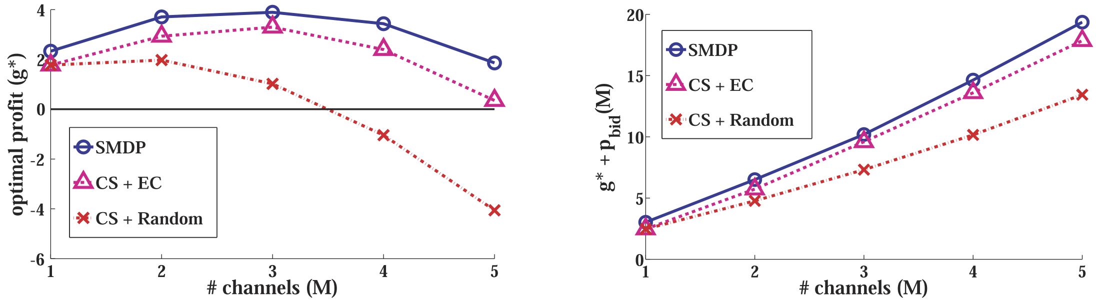
Fig. 7. The state-transition diagrams of the proposed SMDP algorithm, according to user arrivals (shown for $m = 2$).

7. In Section 7.2, we test both exponential and Erlang distributions; otherwise, ON/OFF periods are assumed to be exponentially distributed. For the Erlang case, we consider pdfs of $(\eta^i_{ON})^2 t \cdot e^{-\eta^i_{ON} \cdot t}$ and $(\eta^i_{OFF})^2 t \cdot e^{-\eta^i_{OFF} \cdot t}$, $t > 0$, where $\eta^i_{ON} = 2/E[T^i_{ON}]$ and $\eta^i_{OFF} = 2/E[T^i_{OFF}]$.



(a) Exponential distribution (b) Erlang distribution

Fig. 8. Approximation accuracy of (5) for two types of ON/OFF distributions.



(a) optimal profit g^* (b) $g^* + p_{bid}(M)$

Fig. 9. Comparison of SMDP and two variations of CS in terms of their achieved profits.

$g^* + p_{bid}(M)$. The CS algorithm provides a simple admission control that accepts any arrivals as long as there is room in the WSP's spectrum resources. Since CS is not designed to deal with eviction control, we consider two possible variations of CS: CS with random eviction (denoted by "CS + Random"), and CS with optimal eviction control (denoted by "CS + EC"). At channel vacation, "CS + Random" chooses the users to evict one-by-one through random selection, until the remaining users can fit in the available idle channels. On the other hand, "CS + EC" is derived by applying our SMDP procedure in Sections 4 and 5 by updating $A_1(\mathbf{s})$ in (3) as

$$A_1(\mathbf{s}) = \left\{ \mathbf{a} : \; a_k = \begin{cases} 0 & \text{if } (\mathbf{n} + \mathbf{u}_k)^{\mathrm{T}}\mathbf{B} > mC, \\ 1 & \text{otherwise.} \end{cases} \right\}.$$

Fig. 9 shows that SMDP always achieves more profit than CS. In the tested scenario, SMDP achieves up to 44 percent more profit than "CS + Random" and up to 22.5 percent more profit than "CS + EC", in terms of $g^* + p_{bid}(M)$.[8] Comparison of "CS + EC" and "CS + Random" shows that the performance of CS is enhanced significantly by employing the optimal eviction control. In addition, the performance gap between the optimal SMDP and "CS + EC" is much smaller than the gap between "CS + EC" and "CS + Random", suggesting that eviction control makes more impact on the achieved profit than the admission control does. It also shows that "CS + EC" coincides with "CS + Random" at $M = 1$ since there exists only one possible eviction control at channel vacation (i.e., evict all users), and the performance gap between two becomes more significant as M grows. This implies that the effect of eviction control becomes dominant as the number of leased channels increases, due to the time-varying spectrum availability.

8. The reason why we consider $g^* + p_{bid}(M)$ is to eliminate the effect of the bidding price for fair comparison, focusing on the profit achievement solely by the optimal control, since $p_{bid}(M)$ is given and thus cannot be optimized.

7.3.2 *Impact of Number of Leased Channels* M

In Fig. 9a, one can see that g^* varies with M, having a peak value at $M = 3$. This phenomenon stems from the tradeoff between the revenue generated by customers (i.e., $g^* + p_{bid}(M)$) and the bidding cost (i.e., $p_{bid}(M)$), because 1) a larger M generates more revenue and less reimbursement cost due to more room available to accommodate user arrivals, but 2) the gain will eventually be saturated due to the bounded user population, and therefore, leasing more channels than necessary becomes counterproductive, considering that $p_{bid}(M)$ grows faster than proportionally to M as assumed in Section 3.2. Therefore, finding a proper M is essential to profit maximization which can be achieved by determining and comparing $g^*(M)$ for various M using the proposed SMDP algorithm.

7.3.3 *Impact of Service Tariff* $\mathbf{p}$

Due to the price-dependent arrival rate (i.e., $\lambda_k(p_k) = \lambda_k^{max} e^{-p_k}$), varying p_k may produce a different amount of profit. To show its impact on g^*, we evaluated the case of $M = 3$ and $K = 2$ while varying $\mathbf{p}$ such that $0.5 \leq p_1, p_2 \leq 3.50$. Fig. 10 plots the resulting g^*. It can be seen that as either p_1 or p_2 (or both) gets closer to 0.5 or 3.5, the resultant profit decreases dramatically due to the bounded user arrivals by λ_k^{max} (the case of 0.5) and decrease of the user arrival rate by e^{-p_k} (the case of 3.5), respectively. As a result, the profit function becomes concave, where the largest

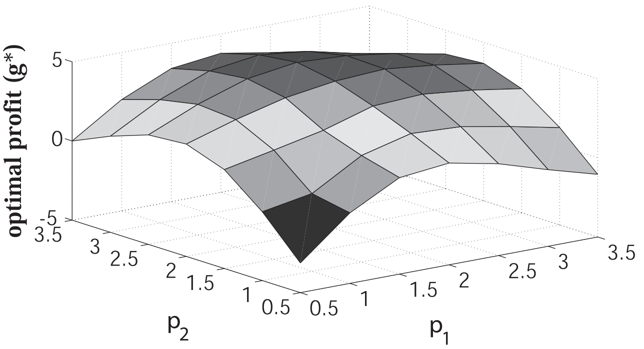


Fig. 10. Optimal profit with various end-user pricing.

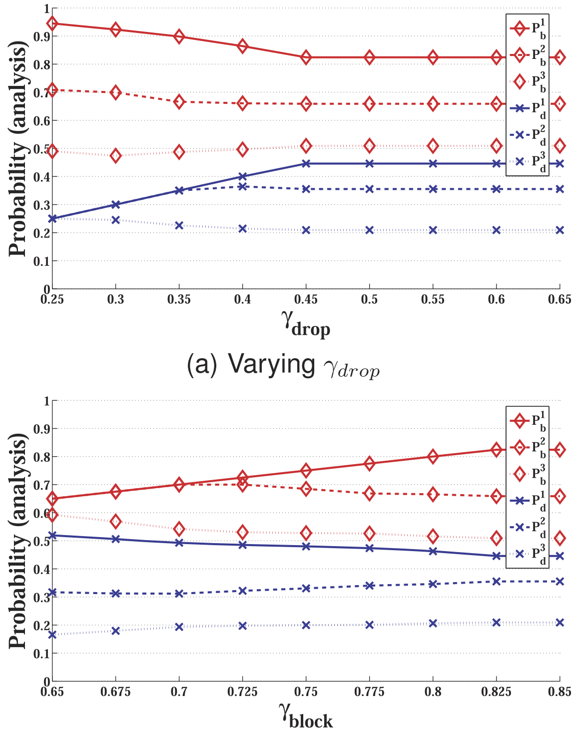
profit can be attained at $\mathbf{p} = (2.0, 2.5)$. Therefore, a WSP must consider the impact of its pricing policy on the overall profit, for which research on market demands and customer statistics should be helpful to find the price-arrival rate relationship.

7.4 Tradeoffs between Two QoS Constraints

Fig. 11 plots the tradeoff between P_b and P_d. To show the relationship between the two, we first fix $\gamma_{block} = 1.0$ and vary γ_{drop} in Fig. 11a. Similarly, in Fig. 11b, we fix $\gamma_{drop} = 1.0$ and vary γ_{block}. As γ_{drop} decreases, the QoS requirement on P_d becomes stricter so that more in-service users may be protected from eviction at channel vacation. For this, user arrivals must be blocked more often since the longer stay of in-service users implies less idle resources for newly arriving users. Thus, P_b increases to compensate for the decrease of P_d which can be observed from Figs. 11a and 11b, more apparently at class 1. By contrast, as γ_{block} decreases, the QoS requirement on P_b becomes stricter so that less users may be rejected upon their arrival. To accommodate more users, the WSP has to reserve more room by evicting more users at channel vacation, thus increasing P_d. This phenomenon is clearly seen in Fig. 11b at class 1.

8 Conclusion

In this paper, we proposed the (sub)optimal admission and eviction control of CR users to maximize a WSP's profit at a CR hotspot. The problem was modeled as an SMDP, and an LP algorithm was proposed to derive the optimal actions. The two QoS constraints—user blocking and dropping probabilities—have also been considered to strike a balance between profit maximization and user satisfaction. We also introduced two types of prioritized user control to enable differentiated service provisioning. The proposed LP algorithm is shown to outperform the CS algorithm, and its sensitivity to the number of channels and the chosen pricing policy has been studied. In future, we would like to extend this problem to the services that allow degradation of in-service users' QoS for dynamic adaptation to various commercial services. That is, we would like to degrade service quality of certain customers to fit into time-varying spectrum availability rather than evicting customers from the service. In such a scenario, customers can be categorized as either quality- or price-sensitive so that different types of control can be applied.



(a) Varying γ_{drop}

(b) Varying γ_{block}

Fig. 11. The tradeoffs between P_b and P_d.

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References

[1] H. Kim and K.G. Shin, "Optimal Admission and Eviction Control of Secondary Users at Cognitive Radio HotSpots," *Proc. IEEE CS Sixth Ann. Conf. Sensor, Mesh, and AdHoc Comm. and Networks (SECON),* June 2009.

[2] M.A. McHenry, "NSF Spectrum Occupancy Measurements Project Summary," *Shared Spectrum Company Report,* Aug. 2005.

[3] S. Deb, V. Srinivasan, and R. Maheshwari, "Dynamic Spectrum Access in DTV Whitespaces: Design Rules, Architecture, and Algorithms," *Proc. ACM MobiCom,* Sept. 2009.

[4] H. Kim and K.G. Shin, "Understanding Wi-Fi 2.0: From the Economical Perspective of Wireless Service Providers," *IEEE Wireless Comm. Magazine,* vol. 17, no. 4, pp. 41-46, Aug. 2010.

[5] P. Bahl, R. Chandra, T. Moscibroda, R. Murty, and M. Welsh, "White Space Networking with Wi-Fi Like Connectivity," *Proc. ACM SIGCOMM,* Aug. 2009.

[6] J.M. Chapin and W.H. Lehr, "The Path to Market Success for Dynamic Spectrum Access Technology," *IEEE Comm. Magazine,* vol. 45, no. 5, pp. 96-103, May 2007.

[7] R. Kennedy and P. Ecclesine, "IEEE P802.11af Tutorial," IEEE 802.11-10/0742r0, https://mentor.IEEE.org/802.11/dcn/10/11-10-0742-00-0000-p802-11af-tutorial.ppt, July 2010.

[8] J. Jia and Q. Zhang, "Competitions and Dynamics of Duopoly Wireless Service Providers in Dynamic Spectrum Market," *Proc. ACM MobiHoc,* May 2008.

[9] M.M. Buddhikot, "Understanding Dynamic Spectrum Access: Models, Taxonomy and Challenges," *Proc. IEEE Second Symp. Network Frontiers in Dynamic Spectrum Access Networks (DySPAN),* Apr. 2007.

[10] K.W. Ross and D.H.K. Tsang, "Optimal Circuit Access Policies in an ISDN Environment: A Markov Decision Approach," *IEEE Trans. Comm.,* vol. 37, no. 9, pp. 934-939, Sept. 1989.
[11] H. Mutlu, M. Alanyali, and D. Starobinski, "Spot Pricing of Secondary Spectrum Usage in Wireless Cellular Networks," *Proc. IEEE INFOCOM,* Apr. 2008.
[12] B. Ishibashi, N. Bouabdallah, and R. Boutaba, "QoS Performance Analysis of Cognitive Radio-Based Virtual Wireless Networks," *Proc. IEEE INFOCOM,* Apr. 2008.
[13] B. Wang, Z. Ji, and K.J.R. Liu, "Primary-Prioritized Markov Approach for Dynamic Spectrum Access," *Proc. IEEE Symp. Network Frontiers in Dynamic Spectrum Access Networks (DySPAN),* Apr. 2007.
[14] D.R. Cox, *Renewal Theory.* Butler & Tanner Ltd., 1967.
[15] A. Motamedi and A. Bahai, "MAC Protocol Design for Spectrum-Agile Wireless Networks: Stochastic Control Approach," *Proc. IEEE Second Symp. Network Frontiers in Dynamic Spectrum Access Networks (DySPAN),* Apr. 2007.
[16] S. Geirhofer, L. Tong, and B.M. Sadler, "Dynamic Spectrum Access in the Time Domain: Modeling and Exploiting White Space," *IEEE Comm. Magazine,* vol. 45, no. 5, pp. 66-72, May 2007.
[17] H. Kim and K.G. Shin, "Efficient Discovery of Spectrum Opportunities with MAC-Layer Sensing in Cognitive Radio Networks," *IEEE Trans. Mobile Computing,* vol. 7, no. 5, pp. 533-545, May 2008.
[18] H. Kim and K.G. Shin, "In-Band Spectrum Sensing in Cognitive Radio Networks: Energy Detection or Feature Detection?" *Proc. ACM MobiCom,* Sept. 2008.
[19] R. Rajbanshi, Q. Chen, A.M. Wyglinski, G.J. Minden, and J.B. Evans, "Quantitative Comparison of Agile Modulation Techniques for Cognitive Radio Transceivers," *Proc. IEEE Fourth Consumer Comm. and Networking Conf. (CCNC),* Jan. 2007.
[20] Y. Wu, B. Wang, K.J.R. Liu, and T.C. Clancy, "A Multi-Winner Cognitive Spectrum Auction Framework with Collusion-Resistant Mechanisms," *Proc. IEEE Third Symp. Network Frontiers in Dynamic Spectrum Access Networks (DySPAN),* Oct. 2008.
[21] S. Gandhi, C. Buragohain, L. Cao, H. Zheng, and S. Suri, "A General Framework for Wireless Spectrum Auctions," *Proc. IEEE Second Symp. Network Frontiers in Dynamic Spectrum Access Networks (DySPAN),* Apr. 2007.
[22] D. Niyato and E. Hossain, "A Game-Theoretic Approach to Competitive Spectrum Sharing in Cognitive Radio Networks," *Proc. IEEE Wireless Comm. and Networking Conf. (WCNC),* Mar. 2007.
[23] V. Paxson and S. Floyd, "Wide-Area Traffic: The Failure of Poisson Modeling," *IEEE/ACM Trans. Networking,* vol. 3, no. 3, pp. 226-244, June 1995.
[24] S.N. Subramanian and T. Le-Ngoc, "Traffic Modeling in a Multimedia Environment," *Proc. IEEE Canadian Conf. Electrical and Computer Eng. (CCECE/CCGEI),* 1995.
[25] G. Gallego and G.v. Ryzin, "Optimal Dynamic Pricing of Inventories with Stochastic Demand over Finite Horizons," *Management Science,* vol. 40, no. 8, pp. 999-1020, Aug. 1994.
[26] H.C. Tijms, *Stochastic Modelling and Analysis: A Computational Approach.* John Wiley & Sons, 1986.
[27] K.W. Ross and D.H.K. Tsang, "The Stochastic Knapsack Problem," *IEEE Trans. Comm.,* vol. 37, no. 7, pp. 740-747, July 1989.

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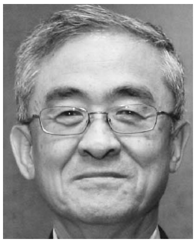

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