

Ergodic Sum Capacity of Spectrum-Sharing Multiple Access with Collision Metric

Mohammad G. Khoshkholgh, Nader Mokari, *Student Member, IEEE*, and Kang G. Shin, *Life Fellow, IEEE*

Abstract—This paper investigates the ergodic sum capacity of a spectrum-shared Multiple Access Channel (MAC). We assume that the secondary service (SS) only knows the channel distribution information (CDI) between its transmitters and the primary receivers. Availability of CDI results in collision incidences at the primary receivers because of conflicting levels of intolerable interference. We introduce the concept of *collision probability constraint* to manage the unexpected QoS degradation of primary service in the secondary resource allocation (RA). This RA problem is inherently difficult to solve and its objective function is not necessarily convex. Two well-known approaches, called *Iterative Approach (IA)* and *Analytical Approach (AA)*, each with several cases/categories, are then used to find solutions. IA solves the problem iteratively by reconstructing convex optimization problems from the original (non-convex) one in a number of iterative loops until the collision probability constraints are satisfied. IA is shown to converge quickly to a suitable solution. Furthermore, by using a control parameter, the system designer can make a tradeoff between the speed of convergence and the ergodic sum capacity. AA, on the other hand, solves the RA problem by suggesting tractable versions of collision probability constraints. Unlike IA, AA does not require extra signaling between transmitters and the base station to tune parameters, thus facilitating the implementation of SS. Our in-depth simulations have shown the proposed approach to yield lower spectral efficiency than IA.

Index Terms—Collision probability, convex optimization, ergodic sum capacity, multiple access channel (MAC), spectrum sharing.

I. INTRODUCTION

ANALYSIS of the information capacity of spectrum sharing reveals not only how much the performance of the licensed bands can be enhanced, but also how appropriate protocols should approach the anticipated spectral efficiency [1], [2]. Spectrum sharing allows secondary/cognitive/unlicensed services to opportunistically access underutilized/white space of the licensed spectrum [3], [4], thus enhancing the overall spectrum utilization. Underlay-, overlay-, and mixed underlay-overlay spectrum sharing are the different ways for the secondary service (SS) to access the primary spectrum (licensed band) [5]. In this paper, we focus on the analysis of the capacity achievable by the SS in the underlay spectrum sharing.

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M. G. Khoshkholgh is with the Simula Research Laboratory, Fornebu, Norway (e-mail: m.g.khoshkholgh@gmail.com).

N. Mokari is with the Wireless Innovation Laboratory, Department of Electrical and Computer Engineering, Tarbiat Modares University, Tehran, Iran (e-mail: nader.nmy@gmail.com).

K. G. Shin is with the Department of Electrical Engineering and Computer Science, The University of Michigan, Ann Arbor, MI, 48109-2121, USA (e-mail: kgshin@eecs.umich.edu).

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To keep the interference inflicted to the primary service (PS) due to the secondary activity below predefined *interference threshold* constraints, the SS must either postpone or give up its transmission activity [6].

The maximum achievable capacity of the underlay spectrum sharing has been studied extensively. The authors of [1], [7] examined the fundamental characteristics of the ergodic capacity of single-link SS in different fading environments. Their results are then extended into Multiple-Input Multiple-Output (MIMO) scenarios [8]. Different combinations of average/peak interference threshold constraints as well as the peak/average transmission power constraint of SS are considered in [9], [10], and the corresponding optimal power-allocation strategies are then derived. They further showed that the combinations, such as average interference threshold and transmission power constraints, yield a higher SS ergodic capacity.

A comprehensive study of resource allocation (RA) in spectrum-sharing settings, including Multiple Access Channel (MAC), Broadcast Channel (BC), and an ad hoc paradigm with MIMO transceivers, has been reported in [2], [11], [12]. Sum-rate maximization in Gaussian cognitive MAC is studied and efficient power allocation with convex relaxation is then derived in [13]. These results were extended to the case of fading channels in [14], [15]. Utilizing the dual decomposition technique, the authors of [14] investigated the optimal power allocation and user selection in MAC and BC to maximize the ergodic sum capacity of the SS in a fading environment subject to different combinations of transmission power and interference threshold constraints.

All of the above-mentioned studies assume that the SS is able to perfectly estimate link between its transmitter and the primary receivers. As pointed out in [16], [17], the SS may, in reality, be unable to accurately track these fading gains which, in turn, leads to spectral-efficiency degradation, unacceptable PS QoS-degradation, and unrealistic capacity prediction. Therefore, there is a strong need for approaches including more meticulous analyses of realistic scenarios. The main contribution of this paper is to analyze the effects of the lack of channel state information (CSI) between the secondary transmitters and the primary receivers on the ergodic sum capacity of the SS in the fading MAC environments. Here, we assume that only channel distribution information (CDI) between the SS transmitters and the PS receivers is available at the SS. Similar to [18], [19], we introduce the statistical version of the interference threshold constraint, called the *collision probability constraint*, to model the spectrum-sharing requirement. The collision probability constraint, (Q, ξ) , man-

ages the collision incidences caused by the SS's intolerable interference to the primary receiver, higher than the interference threshold constraint Q , but below the maximum allowable collision probability ξ .

The MAC RA problem considered in this paper is inherently difficult to deal with mainly because of the stochastic nature of the collision probability constraints. To cope with the vicissitudes inherent in our MAC RA problem and develop important (and practical) suboptimal solutions, we introduce two well-known approaches: *Iterative Approach* (IA) and *Analytical Approach* (AA).

In IA we trade the collision probability constraints for more familiar, easy-to-deal-with constraints—such as average/peak transmission-power constraints or average/peak interference threshold constraints (constructed by utilizing CDI between the secondary transmitters and the primary receivers)—and reconstruct the optimization problems. Fortunately, the reconstructed optimization problems are convex and efficient numerical algorithms can be exploited to quickly find the solutions. Our key idea is that in each iteration, we adjust an appropriate set of peak/average transmission power constraints or peak/average interference threshold constraints until the collision probability constraints at the all PS receivers are met.

In AA we derive mathematically suitable versions of the collision probability constraints by inferring them as peak/average transmission power constraints. However, unlike IA, here we analytically reconstruct suitable interpretations of the collision probability constraints. Interpreted constraints are in general functions of pdf of the channel power gains and the collision probability constraints (Q, ξ) . Here, each secondary transmitter independently construes the collision probability constraints into its own suitable peak/average transmission power constraints. Thus, unlike IA, we do not require extra signaling among the secondary transmitters and the secondary base station to adjust appropriate power/interference constraints. However, this interesting implementation attribute degrades spectral efficiency of the SS. This is anticipated as the collision incidence at every primary receiver results from the aggregated interference of secondary transmitters.

This paper is organized as follows. Section II describes the system model and states the RA problem. Sections III and IV present iterative and analytical approaches to the RA problem, respectively. Section V provides our numerical and simulation results. Finally, Section VI summarizes our main results.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a B Hz wireless flat fading channel with Additive White Gaussian Noise (AWGN). AWGN spectral power is N_0B . The PS consists of J nodes (receivers) where the index set is denoted by $\mathcal{J} = \{1, \dots, J\}$. The SS is made up of a base station and K secondary users communicating with the base station via an uplink channel. $\mathcal{K} = \{1, \dots, K\}$ is the secondary users index set. We denote a $K \times 1$ vector $\mathbf{g}_{ss} = [g_{ss}^1, \dots, g_{ss}^K]^T$ as the vector channel power gain between the secondary users and the base station where T denotes the transpose operator. A random vector \mathbf{g}_{ss} is drawn from the continuous probability density function (pdf) $f_{ss}(\mathbf{g}_{ss})$. Furthermore, a $K \times 1$ vector $\mathbf{g}_{sp}^j = [g_{sp}^{j1}, \dots, g_{sp}^{jK}]^T$ represents

the vector channel power gain between the secondary users and the PS receiver $j \in \mathcal{J}$. For each j , the random vector \mathbf{g}_{sp}^j is drawn according to the continuous pdf $f_{sp}^j(\mathbf{g}_{sp}^j)$. Note that in addition to the independence of \mathbf{g}_{ss} and $\mathbf{g}_{sp}^j, \forall j$, all entries of each vector are assumed to be independent of one another. We assume that the SS base station has perfect CSI of \mathbf{g}_{ss} , but it only has access to the CDI of $\mathbf{g}_{sp}^j, \forall j \in \mathcal{J}$.

To obtain the CSI between the secondary users and the PS, we need approaches including reciprocity and synchronous signaling between SS and PS, which are unrealistic, particularly when there are too many PSs. Note that the PS is usually not designed to collaborate with SS. Furthermore, availability of CSI imposes a new communication burden on the PS. On the other hand, to obtain CDI, the SS needs only to estimate path-loss attenuation and the shadowing gain in the case of Rayleigh fading. In Rician and Nakagami fading, besides path-loss attenuation and shadowing gain, the power of Line-Of-Sight (LOS) component is required. In practice, shadowing varies slowly and its estimation is easier than estimating fast fading fluctuations. In this paper, the main spectrum-sharing constraint is the *collision probability constraint*, (Q^j, ξ^j) , for the primary receiver j . The parameter ξ^j is the allowable collision probability at the primary receiver j . Next, we describe the appropriate RA problem in detail.

A. Problem Formulation

In the underlay spectrum sharing, the SS will want to maximize its spectral efficiency. Nevertheless, the collision probability constraint at the primary receivers prevents the SS from being in the high spectral-efficiency realm. Noting the collision probability constraints, $(Q^j, \xi^j), \forall j$, and power budget constraints of the secondary transmitters $\bar{P}_s^k, \forall k$, and denoting $C(\mathbf{P}(\mathbf{g}_{ss})) = \log \left(1 + \sum_{k=1}^K \frac{g_{ss}^k P_s^k(\mathbf{g}_{ss})}{N_0 B + I_{BS}} \right)$ the RA problem is then formulated as:

$$\mathbf{P} \mathcal{O}_{SM}: \quad C = \max_{\{\mathbf{P}(\mathbf{g}_{ss}) \geq 0\}} \mathbf{E}_{\mathbf{g}_{ss}} [C(\mathbf{P}(\mathbf{g}_{ss}))], \quad (1)$$

$$\text{s.t.} \quad (\text{C1}): \mathbf{E}_{\mathbf{g}_{ss}} [P_s^k(\mathbf{g}_{ss})] \leq \bar{P}_s^k, \quad \forall k \in \mathcal{K}, \quad (2)$$

$$(\text{C2}): \mathbf{P} \left\{ \sum_{k=1}^K g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) > Q^j \right\} \leq \xi^j, \quad \forall j \in \mathcal{J}. \quad (3)$$

Eq. (1) denotes the ergodic sum capacity of SS when the optimal Gaussian code book is used at each secondary user and successive interference cancellation is considered at the base station [20], [21]. The effective Gaussian noise at the base station includes the AWGN and the aggregated imposed interference due to the PS transmissions, I_{BS} [8], [14], [22]. In \mathcal{O}_{SM} a $K \times 1$ vector $\mathbf{P}(\mathbf{g}_{ss}) = [P_s^1(\mathbf{g}_{ss}), \dots, P_s^K(\mathbf{g}_{ss})]^T$ denotes the power allocated to the secondary users. In solving the problem \mathcal{O}_{SM} , one may suffer some drawbacks as follows. We need to evaluate the collision probabilities,¹ so it is necessary to find the pdfs of random variables $\sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss})$,

¹Note that (3) is actually the interference outage probability constraint. Here we refer to (3) as the collision probability constraint following the definition suggested in [3].

$\forall j \in \mathcal{J}$. For the special case of Rayleigh fading, we have²

$$\mathcal{CO}^j = \mathbf{E}_{g_{ss}} \left[\left(\prod_{k=1}^K \frac{1}{\mu_{jk} P_s^k(g_{ss})} \right) \sum_{k=1}^K P_s^k(g_{ss}) \mu_{jk} \times \frac{e^{-\frac{Q^j}{\mu_{jk} P_s^k(g_{ss})}}}{\prod_{n \neq k} \left(\frac{1}{\mu_{jn} P_s^n(g_{ss})} - \frac{1}{\mu_{jk} P_s^k(g_{ss})} \right)} \right],$$

where μ_{jk} , $\forall j, k$ are the mean values of g_{sp}^{jk} . Substituting Eq. (II-A) in \mathcal{O}_{SM} , we get the MAC RA problem with homogeneous constraints (all are identical with respect to the mean operator). Unfortunately, Eq. (II-A) makes the RA problem mathematically intractable. Note that for the other usual wireless channel models such as log-normal shadow and Rician fading, we may not be able to find the pdfs of $\sum_k g_{sp}^{jk} P_s^k(g_{ss})$, $\forall j \in \mathcal{J}$. Although one may take such approaches as stochastic geometry [24], Central Limit Theorem, and chance-constrained programming [25] to approximate the collision probability, their results are still mathematically unsuitable for the MAC RA.³ The other drawback associated with the problem \mathcal{O}_{SM} , due mainly to intractable collision probability constraints, is the lack of convexity/concavity. As a result, an exhaustive numerical search may yield multiple local optima.

Remark 1: To highlight the issue related to the convexity of the problem \mathcal{O}_{SM} , we briefly examine the special case of $J = K = 1$. In this case, the collision probability constraint is $\mathbf{E}_{g_{ss}} \bar{F}_{g_{sp}} \left(\frac{Q}{P_s(g_{ss})} \right) \leq \xi$ where we removed primary and secondary indices for notational simplicity. Note that for random variable X we have $F_X(x) = 1 - \bar{F}_X(x)$. Consider the function $w(z) = \bar{F}_{g_{sp}} \left(\frac{Q}{z} \right)$, $z \geq 0$. The function $v(z) = \frac{Q}{z}$ is convex and $\bar{F}_{g_{sp}}(x)$ is a non-increasing function of $x \geq 0$. So, $w(z)$ is convex if $\bar{F}_{g_{sp}}(x)$ is a concave function. In such a case, \mathcal{O}_{SM} is convex and the optimal solution is obtained by solving the following equation for y

$$\frac{g_{ss}}{N_0 B + I_{BS} + g_{ss} y} - \lambda_1 + \lambda_2 \frac{Q}{y^2} \bar{F}'_{g_{sp}} \left(\frac{Q}{y} \right) = 0.$$

Thus, $P_s^* = \max\{y, 0\}$. Here λ_1 and λ_2 are the Lagrangian multipliers associated with transmission power and collision probability constraints, respectively. When $\bar{F}_{g_{sp}}(x)$ is convex, the RA problem is not convex. As a result, the above solution may not be optimal. In reality, many interesting fading distributions, such as Rayleigh fading, are not concave. In the case of Rayleigh fading, it is easy to verify that $w''(P_s) = \mu_{sp} e^{-\frac{\mu_{sp} Q}{P_s}} \left(\frac{\mu_{sp} Q}{P_s} - 2 \right)$. To ensure the convexity

²For this, we can use the result of [23]: Let X_1, \dots, X_n for a given fixed integer $n \geq 2$ be i.i.d. exponential random variables, each with distinct mean value γ_i . The pdf of random variable $S_n = \sum_i X_i$ is then given as

$$f_{S_n}(x) = \left(\prod_{i=1}^n \gamma_i \right) \sum_{i=1}^n \frac{e^{-\gamma_i x}}{\prod_{k \neq j} (\gamma_k - \gamma_j)}, \quad x > 0.$$

³Note that this difficulty may be mitigated if some common upper-bounds of the collision probabilities are considered. For instance, in the case of a Rayleigh fading channel adopting the Chebyshev inequality, an upper-bound of the collision probability \mathcal{CO}^j is $\mathcal{CO}^j \leq$

$$\mathbf{E}_{g_{ss}} \left[\frac{\sum_{k=1}^K 1/(\mu_{jk} P_s^k(g_{ss}))^2}{\left(Q^j - \sum_{k=1}^K 1/\mu_{jk} P_s^k(g_{ss}) \right)^2} \right] \text{ which is still not easy to work with.}$$

of the optimization problem, we could add an extra power allocation constraint, such as $P_s(g_{ss}) \geq \frac{\mu_{sp} Q}{2}$, $\forall g_{ss}$. The same procedure can also be adopted for other fading distributions.

Instead of solving \mathcal{O}_{SM} directly, we propose approaches that either eliminate constraints (3) or substitute them with affine functions associated with interference threshold constraints. This converts the RA problem to new convex optimization problems. Each reconstructed optimization problem is mathematically tractable and efficient, and powerful convex programming can be applied to find the solutions.

We adopt two different approaches to solve \mathcal{O}_{SM} : (i) *Iterative Approach* (IA) that works iteratively and (ii) *Analytical Approach* (AA) that solves the RA problem analytically. More precisely, in IA the key parameters for reconstructing the optimization problem are obtained iteratively. However, in AA an analytical procedure is adopted to substitute the problem \mathcal{O}_{SM} with reconstructed convex optimization problems.

A main advantage of AA over IA is that each secondary transmitter can build its own reconstructed optimization problem without sharing extra information regarding the collision probabilities with other transmitters and the base station. However, IA requires some level of signaling between the base station and the secondary transmitters to concoct the proper reconstructed optimization problems.

In the rest of this paper, each of IA and AA is detailed and the corresponding power allocation strategy is derived.

III. ITERATIVE APPROACH (IA)

IA is to iteratively adjust artificial power constraints and/or interference threshold constraints so that, after a sufficient number of iterations, all primary receivers have their collision probability constraints satisfied. In each iteration, it solves a simpler optimization problem constructed from problem \mathcal{O}_{SM} , which we call a *reconstructed optimization problem*. The reconstructed optimization problem is easier to solve and mathematically more tractable because the collision probability constraints (3) in \mathcal{O}_{SM} are either vanished completely or substituted with affine power or interference functions. Moreover, the reconstructed optimization problem is convex, so powerful and efficient numerical algorithms such as the Ellipsoid method can be employed to find a solution. After finding a proper power allocation in each iteration, the base station evaluates the collision probabilities using (II-A) or the Monte Carlo simulation. Depending on the thus-obtained collision probabilities and according to the developed algorithm, some parameters—artificial power/interference constraints—will be adjusted, and then the algorithm repeats itself. This will continue until all the collision probabilities fall below the corresponding collision probability constraints.

Next, we elaborate on different cases and develop appropriate algorithms in each case that converge to a suitable power allocation.⁴

⁴Recall that in each iteration of IA the collision probabilities can be evaluated either via the Monte Carlo simulation or the approach in (II-A). If a closed-form solution of the reconstructed optimization problem can be derived, we may apply the approach in (II-A). However, the SS can always adopt an off-line Monte Carlo simulation to find the collision probabilities since the SS has access to the CDI between users and all primary receivers.

A. Iterative Appropriate Maximum Transmission Power Adaption (IAMTPA)

In IAMTPA, in each iteration, the reconstructed optimization problem encompasses neither the collision probability constraints nor versions of interference threshold constraints. Instead, transmission power constraints are adapted iteratively to satisfy the collision probability constraints. IAMTPA is divided further into A-IAMTPA and P-IAMTPA, which are detailed next.

1) *A-IAMTPA*: Let $n = 1, 2, \dots$ be the iteration count and $\theta(n) \in [0, 1]$ be a scaling parameter. In A-IAMTPA, $\bar{P}_s^k(n)$ —the adjusted k -th maximum transmission power constraint at the n -th iteration—is yielded to be $\bar{P}_s^k(n) = \theta(n)\bar{P}_s^k(n-1) \leq \bar{P}_s^k, \forall k \in \mathcal{K}$. We further assume that $\bar{P}_s^k(1) = \bar{P}_s^k$. Then, A-IAMTPA works as follows.

Step 1: Reconstruct the optimization problem at the n -th iteration as:

$$\mathbf{PO}_{SM}^{A-IAMTPA}.$$

$$C = \max_{\{\mathbf{P}(\mathbf{g}_{ss}) \geq 0\}} \mathbf{E}_{\mathbf{g}_{ss}} [C(\mathbf{P}(\mathbf{g}_{ss}))], \quad (4)$$

$$\text{s.t. } \mathbf{E}_{\mathbf{g}_{ss}} [P_s^k(\mathbf{g}_{ss})] \leq \bar{P}_s^k(n), \quad \forall k \in \mathcal{K}, \quad (5)$$

that is a convex optimization problem, making a local solution the global one. Problem $\mathbf{O}_{SM}^{A-IAMTPA}$ is similar to those considered in [20] and [21]. Solving $\mathbf{O}_{SM}^{A-IAMTPA}$ at each iteration amounts to finding the maximum ergodic sum capacity of a service without spectrum-sharing. The optimal RA strategy has been shown in [21] to be the well-known Time Division Multiple Access (TDMA). At the n -th iteration the optimal power-allocation strategy is

$$P_s^{k(n)}(\mathbf{g}_{ss}) = \begin{cases} \left(\frac{1}{\lambda^k} - \frac{N_0 B + I_{BS}}{g_{ss}^k} \right)^+ & \text{if } g_{ss}^k > \frac{\lambda^k}{\lambda^l} g_{ss}^l, \\ 0 & \text{otherwise,} \end{cases} \quad (6)$$

where λ^k are the Lagrangian multipliers associated with the transmission power constraints.

Step 2: Using the optimal power-allocation strategy, compute the collision probabilities at the n -th iteration, $\mathcal{CO}^j(n)$.

Step 3: Compute $\mathcal{D}^j(n) = |\mathcal{CO}^j(n) - \xi^j|$ for $\forall j \in \mathcal{J}$. If $\mathcal{D}^j(n) \leq \epsilon, \forall j \in \mathcal{J}$, then $P_s^{k(n)}(\mathbf{g}_{ss})$ are the optimal power allocations; otherwise, $\theta(n)$ is obtained from $\theta(n) = 1 - \max_j \{\mathcal{D}^j(n)\}$. Adjust the transmission power constraints as $\bar{P}_s^k(n+1) = \theta(n)\bar{P}_s^k(n)$. Go to Step 2, and repeat the algorithm.

For all secondary transmitters, the corresponding process, $\bar{P}_s^k(n)$, is decreasing since $\theta(n) \in [0, 1]$. Therefore, $\mathcal{CO}^j(n)$ will be decreased by increasing n for all primary receivers. Equivalently, $\theta(n)$ is an increasing function. As $n \rightarrow \infty$, $\theta(n) \rightarrow 1$, meaning the A-IAMTPA algorithm's convergence.

Remark 2: In A-IAMTPA we can control the speed of the convergence via $\theta(n) = (1 - \max_j \mathcal{D}^j(n))^\Theta$ where $\Theta \geq 1$. In

fact, for $\Theta > 1$ the amount of $\bar{P}_s^k(n)$ will be adjusted faster, thus making the algorithm converge quicker. However, this faster convergence comes at the cost of smaller ergodic sum capacity. This is mainly because for $\Theta > 1$ the algorithm

⁵ $\epsilon \in \mathbb{R}^+$ is a sufficiently small number.

jumps over some feasible power allocations that may satisfy the collision probability constraints.

2) *P-IAMTPA*: As in A-IAMTPA, the collision probability constraints are completely ignored in each iteration. Instead of iteratively adjusting the transmission power constraints, however, it adopts an auxiliary set of peak transmission power constraints. The suitable values of peak transmission power constraints are determined by running P-IAMTPA.

Let $P_s^k \forall k \in \mathcal{K}$ denote the auxiliary peak transmission power constraints, and n the iteration count. Like in A-IAMTPA, we define the parameter $\theta(n) \in [0, 1]$. Then, P-IAMTPA works as follows.

Step 1: The initial values of the peak transmission power constraints, $P_s^k(n = 1)$, are determined from $P_s^k(1) = \pi^k \frac{\max_j Q^j}{\min_j 1/\mu_{jk}}$ where $\pi^k, \forall k \in \mathcal{K}$, are arbitrary real, large parameters. The peak power $P_s^k(1)$ is determined as follows. Use of the peak transmission power $P_s^k(1)$ must ensure the collision probabilities at all primary receivers to be larger than the collision probability constraints. We first assume that the secondary transmitter k is the only transmitter (while others stay silent) and transmits with its peak power $P_s^k(1)$. Due to this allocated transmission power, the interference incurred to the j -th primary receiver is equal to $P_s^k(1)g_{sp}^{jk}$ which should be greater than the interference threshold $\max_j Q^j$. By doing this for all other secondary transmitters, we can obtain $P_s^k(1) = \frac{\max_j Q^j}{\min_j g_{sp}^{jk}}$. However, since the SS only has access to

$\mu_{jk}, \forall j \in \mathcal{J}, k \in \mathcal{K}$, we can build $P_s^k(1) = \frac{\max_j Q^j}{\min_j 1/\mu_{jk}}$. The initial values are determined by observing the mean values of the channel power gains, so we need to scale the $\frac{\max_j Q^j}{\min_j 1/\mu_{jk}}$ such that the unknown fading realizations are also appropriately accounted for. Since the fading gain in wireless channels often fluctuates with 30–40 dB, the parameters π^k can also be chosen in this range. The thus-chosen π^k 's are large enough in each iteration to satisfy $P_s^k(n+1) = \theta(n)P_s^k(n) \leq P_s^k(n)$.

Step 2: The reconstructed optimization problem in the n -th iteration is then formulated as:

$$\mathbf{PO}_{SM}^{P-IAMTPA}.$$

$$C = \max_{\{0 \leq \mathbf{P}(\mathbf{g}_{ss}) \leq P_s^k, \forall k \in \mathcal{K}\}} \mathbf{E}_{\mathbf{g}_{ss}} [C(\mathbf{P}(\mathbf{g}_{ss}))], \quad (7)$$

$$\text{s.t. } \mathbf{E}_{\mathbf{g}_{ss}} [P_s^k(\mathbf{g}_{ss})] \leq \bar{P}_s^k, \quad \forall k \in \mathcal{K}. \quad (8)$$

This optimization problem is almost the same as those in [14], [26]. Note that $\mathbf{O}_{SM}^{P-IAMTPA}$ is a convex optimization problem. Thus, solving the dual optimization $\min_{\{\lambda\} \geq 0} D(\{\lambda\})$ will find the optimal solution. Introducing the Lagrangian multiplier $\{\lambda\}$ associated with constraints (8), the dual function is $D(\{\lambda\}) = \mathbf{E}_{\mathbf{g}_{ss}} [\hat{D}(\{\lambda\})] + \sum_{k=1}^K \lambda^k \bar{P}_s^k$ where $\hat{D}(\{\lambda\})$ is obtained as

$$\hat{D}(\{\lambda\}) = \max_{\{P_s^k \leq P_s^k(n), \forall k \in \mathcal{K}\}} \log \left(1 + \frac{\sum_{k=1}^K g_{ss}^k P_s^k}{N_0 B + I_{BS}} \right) - \sum_{k=1}^K \lambda^k P_s^k,$$

where we drop g_{ss} for simplicity. To solve this convex optimization problem, we follow the same approach in [14]. Assume a permutation ϑ over \mathcal{K} . Let set $\bar{\mathcal{K}} \subseteq \mathcal{K}$ denote the secondary users who transmit with positive power where $\bar{\mathcal{K}} = \{\vartheta(1), \dots, \vartheta(|\bar{\mathcal{K}}|)\}$. The permutation ϑ is defined as $\frac{g_{ss}^{\vartheta(i)}}{\lambda^{\vartheta(i)}} \geq \frac{g_{ss}^{\vartheta(j)}}{\lambda^{\vartheta(j)}}$, $i < j$. The value of $|\bar{\mathcal{K}}|$ is also obtained from $|\bar{\mathcal{K}}| = \max_x (x \frac{g_{ss}^{\vartheta(x)}}{\lambda^{\vartheta(x)}} - N_0 B - I_{BS} - \sum_{r=1}^{x-1} g_{ss}^{\vartheta(r)} P_s^{\vartheta(r)}(n))^+$ and hence the power obtains from the equation $P_s^{\vartheta(a)(n)}$ at the top of the next page.

Step 3: Evaluate the collision probabilities $\mathcal{CO}^j(n)$, $\forall j \in \mathcal{J}$.

Step 4: Compute $\mathcal{D}^j(n) = |\mathcal{CO}^j(n) - \xi^j|$ for $\forall j \in \mathcal{J}$. If $\mathcal{D}^j(n) \leq \epsilon$, $\forall j \in \mathcal{J}$, then $P_s^k(n)$ is the suitable peak transmission power constraints; otherwise, by obtaining $\theta(n) = 1 - \max_j \{\mathcal{D}^j(n)\}$ the transmission power constraints are scaled as $P_s^k(n+1) = \theta(n)P_s^k(n)$. Go back to Step 2, and repeat the procedure.

Note that the processes $P_s^k(n)$ are expected to be decreasing for all secondary transmitters. The lower the peak transmission power constraints, the smaller the level of the aggregated interference at each primary receiver. Therefore, as $n \rightarrow \infty$, $\theta(n) \rightarrow 1$, thus P-IAMTPA converges. Moreover, by defining $\theta(n) = (1 - \max_j \{\mathcal{D}^j(n)\})^\Theta$ for $\Theta \geq 1$, we can control the speed of the algorithm's convergence.

Another approach to dealing with the collision probability constraints is to toy with versions of interference threshold constraints in each iteration. One may conjecture the faster convergence of IAITA than IAMTPA.

B. Iterative Appropriate Interference Threshold Adaption (IAITA)

Under IAITA, in each iteration, the collision probability constraints are regarded as some versions of the peak/average interference threshold constraints and then the reconstructed optimization problem is formulated. The values of the peak/average interference thresholds are iteratively adapted until the collision probability constraints are met. IAITA is also divided into two separate branches regarding the corresponding reconstructed optimization problem: Average IAITA (A-IAITA) and Peak IAITA (P-IAITA).

1) A-IAITA: In this case, we replace the collision probability constraints with the average versions of interference threshold constraints. Considering the paucity of accurate CSI in links g_{sp}^j , $\forall j \in \mathcal{J}$, the average interference threshold constraints in the reconstructed optimization problem are formulated with $\mathbf{E}_{g_{ss}} \left[\sum_k P_s^k(g_{ss}) / \mu_{jk} \right] \leq \tilde{Q}^j$ where \tilde{Q}^j are auxiliary parameters and determined by the algorithm. Let $\tilde{Q}^j(n)$, $\forall j \in \mathcal{J}$, denote the auxiliary average interference threshold constraints where n is the iteration count. As in IAMTPA, we define the parameter $\theta(n) \in [0, 1]$. Then, A-IAITA works as follows.

Step 1: The initial values of average interference thresholds, $\tilde{Q}^j(1)$, $\forall j \in \mathcal{J}$, are established by $\tilde{Q}^j(0) = \pi^j \max_j Q^j$ where π^j s are large enough positive numbers. Again, these parameters need to be so chosen that the processes $\tilde{Q}^j(n)$ may maintain $\tilde{Q}^j(n+1) = \theta(n)\tilde{Q}^j(n) \leq \tilde{Q}^j(n)$. The parameters π^j capture the effect of small-scale fading fluctuation since we constructed the interference threshold constraint by considering the mean channel power gains.

Step 2: Formulate the reconstructed optimization problem as:

$\mathbf{PO}_{S, \mathcal{M}}^{A-IAITA}$:

$$C = \max_{\{P_s(g_{ss}) \geq 0\}} \mathbf{E}_{g_{ss}} [C(P(g_{ss}))], \quad (6)$$

$$\text{s.t.} \quad (C1), \mathbf{E}_{g_{ss}} \left[\sum_{k=1}^K P_s^k(g_{ss}) / \mu_{jk} \right] \leq \tilde{Q}^j(n), \quad \forall j \in \mathcal{J}, (7)$$

This optimization problem is a convex optimization and its solution can be found using the results in [14]. In each fading realization only a secondary transmitter k transmits and the others stay silent. Denoting λ^k and δ^j as the Lagrangian multipliers associated with constraints (C1) and (7), respectively, the solution of the reconstructed optimization problem is then obtained from the second equation $P_s^k(n)$ at the top of the next page.

Step 3: Evaluate the collision probabilities $\mathcal{CO}^j(n)$, $\forall j \in \mathcal{J}$.

Step 4: Compute $\mathcal{D}^j(n) = |\xi^j - \mathcal{CO}^j(n)|$, $\forall j \in \mathcal{J}$. If $\mathcal{D}^j(n) \leq \epsilon$, $\forall j \in \mathcal{J}$, then $\tilde{Q}^j(n)$ is the suitable interference threshold constraints; otherwise, by obtaining $\theta(n) = 1 - \max_j \{\mathcal{D}^j(n)\}$ the interference threshold constraints are scaled as $\tilde{Q}^j(n+1) = \theta(n)\tilde{Q}^j(n)$. Go back to Step 2, and repeat the procedure.

The convergence of A-IAITA can also be proved by following the same reasoning as IAMTPA.

2) P-IAITA: In this case, the collision probability constraints are substituted with peak interference threshold constraints in each iteration loop. Similar to A-IAITA, we utilize the CDI of channel-power gain to establish the peak interference threshold constraints. The interference threshold constraints in the reconstructed optimization problem are formulated via $\sum_k P_s^k(g_{ss}) / \mu_{jk} \leq \tilde{Q}^j$ where \tilde{Q}^j are auxiliary parameters whose values are obtained using the developed algorithm.

Let $\hat{Q}^j(n)$, $\forall j \in \mathcal{J}$, denote the auxiliary peak interference threshold constraints where n is the iteration count (and $\theta(n) \in [0, 1]$). Then, the algorithm works as follows.

Step 1: The initial peak interference thresholds, $\hat{Q}^j(1)$, $\forall j \in \mathcal{J}$, are determined as $\hat{Q}^j(1) = \pi^j \max_j Q^j$ where π^j are large enough numbers so that the processes $\hat{Q}^j(n)$ satisfy $\hat{Q}^j(n+1) = \theta(n)\hat{Q}^j(n) \leq \hat{Q}^j(n)$.

Step 2: Formulate the reconstructed optimization problem as:

$\mathbf{PO}_{S, \mathcal{M}}^{P-IAITA}$:

$$C = \max_{\{P_s(g_{ss}) \geq 0\}} \mathbf{E}_{g_{ss}} [C(P(g_{ss}))],$$

$$\text{s.t.} \quad (C1), \sum_{k=1}^K P_s^k(g_{ss}) / \mu_{jk} \leq \hat{Q}^j(n), \quad \forall j \in \mathcal{J}.$$

This is a convex optimization problem and thus, the dual decomposition method can be used to find a solution. Let $\{\lambda\}$ be the Lagrangian multipliers associated with (C1), then the dual problem is $\min_{\{\lambda\} \geq 0} D(\{\lambda\})$ where the dual function $D(\{\lambda\})$ is $D(\{\lambda\}) = \mathbf{E}_{g_{ss}} \left[\hat{D}(\{\lambda\}) \right] + \sum_{k=1}^K \lambda^k \bar{P}_s^k$. $\hat{D}(\{\lambda\})$ is obtained by solving the following convex optimization

$$P_s^{\vartheta(a)(n)}(\mathbf{g}_{ss}) = \begin{cases} P_s^{\vartheta(a)}(n), & \text{if } a > |\bar{\mathcal{K}}| \\ \min \left\{ P_s^{\vartheta(a)}(n), \left(\frac{g_{ss}^{\vartheta(|\bar{\mathcal{K}}|)}}{\lambda^{\vartheta(|\bar{\mathcal{K}}|)}} - N_0B - I_{NS} - \sum_{r=1}^{|\bar{\mathcal{K}}|-1} g_{ss}^{\vartheta(|\bar{\mathcal{K}}|)} P_s^{\vartheta(|\bar{\mathcal{K}}|)}(n) \right) \right\}, & \text{if } a = |\bar{\mathcal{K}}| \\ 0, & \text{otherwise.} \end{cases}$$

$$P_s^k(n)(\mathbf{g}_{ss}) = \begin{cases} \left(\frac{1}{\lambda^k + \sum_{j=1}^J \delta^j / \mu_{jk}} - \frac{N_0B + I_{BS}}{g_{ss}^k} \right)^+ & \text{if } \frac{g_{ss}^k}{\lambda^k + \sum_{j=1}^J \delta^j / \mu_{jk}} > \frac{g_{ss}^l}{\lambda^l + \sum_{j=1}^J \delta^j / \mu_{jl}}, \\ 0 & \text{otherwise.} \end{cases}$$

problem:

$$\begin{aligned} \dot{D}(\{\lambda\}) &= \max_{P \geq 0} \log \left(1 + \frac{\sum_{k=1}^K g_{ss}^k P_s^k}{N_0B + I_{BS}} \right) - \sum_{k=1}^K \lambda^k P_s^k, \\ \text{s.t.} \quad & \sum_{k=1}^K P_s^k(\mathbf{g}_{ss}) / \mu_{jk} \leq \tilde{Q}^j(n), \quad \forall j \in \mathcal{J}. \end{aligned}$$

Note that in this case there is no closed-form formula for the power solution [14]. Efficient algorithms like the Ellipsoid method or the algorithm in [26] can be utilized to numerically find an optimal solution.

Step 3: Evaluate the collision probability $\mathcal{CO}^j(n)$, $\forall j \in \mathcal{J}$.

Step 4: Compute $\mathcal{D}^j(n) = |\xi^j - \mathcal{CO}^j(n)|$, $\forall j \in \mathcal{J}$. If $\mathcal{D}^j(n) \leq \epsilon$, $\forall j \in \mathcal{J}$, then $\hat{Q}^j(n)$ is a suitable interference threshold constraint; otherwise, by obtaining $\theta(n) = 1 - \max_j \{\mathcal{D}^j(n)\}$, the transmission power constraints are scaled as $\hat{Q}^j(n+1) = \theta(n)\hat{Q}^j(n)$. Go back to Step 2, and repeat the procedure.

We can draw the same conclusion as A-IAITA for the existence and speed of convergence.

As in IA, we need signaling between the secondary base-station and transmitters to share adjustable collision-probability parameters. By contrast, in AA each secondary transmitter establishes its own interpretation of collision-probability constraints, thus reducing the overall computational complexity.

IV. ANALYTICAL APPROACH (AA)

In this section the collision probability constraints are construed as peak/average SS transmission power constraints. The values of peak/average transmission powers are, in general, functions of the pdf of the wireless channel fading g_{sp}^j and the values of (Q^j, ξ^j) , $\forall j \in \mathcal{J}$. Although only the case of Rayleigh fading is analyzed below, similar results can also be derived for the other interesting fading distributions but we omit them due to page limit. The results derived in other fading situations will, however, strongly depend on the pdf convexity/concavity. In what follows, different categories of AA are studied.

A. Appropriate Maximum Average Transmission Power Adaptation (AMATPA)

Let us consider the collision probability at PS j . One may suggest an upper bound of the collision probability at receiver

j as

$$\mathcal{CO}^j \leq \mathbf{E}_{\mathbf{g}_{ss}} \left[\mathbf{P} \left\{ K \max_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) > Q^j | \mathbf{g}_{ss} \right\} \right]. \quad (6)$$

We fix j and define g_j as $g_j = \max_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss})$ for a given fading state \mathbf{g}_{ss} . By definition,

$$\mathbf{P} \left\{ \max_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) > \frac{Q^j}{K} | \mathbf{g}_{ss} \right\} = \int_{\frac{Q^j}{K}}^{\infty} f_{g_j}(g_j) dg_j, \quad (7)$$

where $f_{g_j}(g_j)$ is

$$f_{g_j}(g_j) = \sum_{k=1}^K \frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} e^{-\frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} g_j} \prod_{\hat{k} \neq k} \left(1 - e^{-\frac{\mu_{j\hat{k}}}{P_s^{\hat{k}}(\mathbf{g}_{ss})} g_j} \right).$$

Substituting $f_{g_j}(g_j)$ into (7), one can derive an upper bound of the left-hand side of (7) as

$$\begin{aligned} & \sum_{k=1}^k \frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} \int_{\frac{Q^j}{K}}^{\infty} e^{-\frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} g_j} \prod_{\hat{k} \neq k} \left(1 - e^{-\frac{\mu_{j\hat{k}}}{P_s^{\hat{k}}(\mathbf{g}_{ss})} g_j} \right) dg_j \\ & \leq \sum_{k=1}^k \frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} \int_{\frac{Q^j}{K}}^{\infty} e^{-\frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} g_j} dg_j = \sum_{k=1}^k e^{-\frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} \frac{Q^j}{K}}. \quad (8) \end{aligned}$$

One may then suggest a new collision probability constraint at the primary receiver j as:

$$\mathbf{E}_{\mathbf{g}_{ss}} \left[\sum_{k=1}^k e^{-\frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} \frac{Q^j}{K}} \right] \leq \xi^j. \quad (9)$$

Substituting (9) in the collision probability constraint in Problem \mathcal{O}_{SM} is reduced to:

$$\mathbf{PO}_{SM}^{AMATPA}:$$

$$C = \max_{\{P_s(\mathbf{g}_{ss}) \geq 0\}} \mathbf{E}_{\mathbf{g}_{ss}} [C(\mathbf{P}(\mathbf{g}_{ss}))], \quad (10)$$

$$\text{s.t.} \quad (\text{C1}), \mathbf{E}_{\mathbf{g}_{ss}} \left[\sum_{k=1}^k e^{-\frac{\mu_{jk}}{P_s^k(\mathbf{g}_{ss})} \frac{Q^j}{K}} \right] \leq \xi^j, \quad \forall j \in \mathcal{J}. \quad (11)$$

The problem $\mathcal{O}_{SM}^{AMATPA}$ is convex if $P_s^k(\mathbf{g}_{ss}) < \frac{\mu_{jk} Q^j}{2K} \forall j, k$, so the dual decomposition technique can optimally solve it. For the case when $P_s^k(\mathbf{g}_{ss}) \geq \frac{\mu_{jk} Q^j}{2K}$ for some j, k , the problem has different local solutions. By solving the dual problem $\min_{\{\lambda\} \geq 0, \{\omega\} \geq 0} D(\{\lambda\}, \{\omega\})$ where $\{\lambda\}$ and $\{\omega\}$ are the Lagrangian multipliers associated with constraints (C1)

and (11), respectively, we can find a solution of problem $\mathcal{O}_{SM}^{AMATPA}$. The dual function, $D(\{\lambda\}, \{\omega\})$, is formulated as:

$$D(\{\lambda\}, \{\omega\}) = \mathbf{E}_{g_{ss}} \left[\dot{D}(\{\lambda\}, \{\omega\}) \right] + \sum_{k=1}^K \lambda^k \bar{P}_s^k + \sum_{j=1}^J \omega^j \xi^j, \quad (12)$$

where $\dot{D}(\{\lambda\}, \{\omega\})$ is

$$\begin{aligned} \dot{D}(\{\lambda\}) &= \max_{P \geq 0} \log \left(1 + \frac{\sum_{k=1}^K g_{ss}^k P_s^k}{N_0 B + I_{BS}} \right) - \sum_{k=1}^K \lambda^k P_s^k \\ &\quad - \sum_{j=1}^J \omega^j \sum_{k=1}^K e^{-\frac{\mu_{jk} Q^j}{P_s^k K}}. \end{aligned} \quad (13)$$

Introducing new Lagrangian multipliers $\{\delta\}$ associated with the power allocation assumption, the KKT conditions become

$$\begin{aligned} &\frac{g_{ss}^k}{N_0 B + I_{BS} + \sum_{l=1}^K g_{ss}^l P_s^{*l}} + \delta^{*k} \\ &= \lambda^k + \sum_{j=1}^J \omega^j \frac{\mu_{jk} Q^j}{K (P_s^{*k})^2} e^{-\frac{\mu_{jk} Q^j}{P_s^{*k} K}}, \quad \delta^{*k} P_s^{*k} = 0. \end{aligned} \quad (14)$$

Suppose that all secondary transmitters except k_1 and k_2 are silent, i.e., $\delta^{*k_1} = 0$ and $\delta^{*k_2} = 0$, and others are strictly positive. Let β denote $g_{ss}^{k_1} P_s^{*k_1} + g_{ss}^{k_2} P_s^{*k_2}$, then the KKT conditions reduce to

$$\begin{aligned} \beta + N_0 B + I_{BS} &= \frac{g_{ss}^{k_1}}{\lambda^{k_1} + \sum_{j=1}^J \omega^j \frac{\mu_{jk_1} Q^j}{K (P_s^{*k_1})^2} e^{-\frac{\mu_{jk_1} Q^j}{P_s^{*k_1} K}}} \\ &= \frac{g_{ss}^{k_2}}{\lambda^{k_2} + \sum_{j=1}^J \omega^j \frac{\mu_{jk_2} Q^j}{K (P_s^{*k_2})^2} e^{-\frac{\mu_{jk_2} Q^j}{P_s^{*k_2} K}}}. \end{aligned}$$

Since the channel power gains are independent, the above equality holds with probability 0. As a result, in each fading realization, only one user, denoted by index k , transmits and other transmitters stay silent. Noting that $\lim_{x \rightarrow \infty} x^2 e^{-x} = 0$, we rewrite the KKT conditions for user k and users $l \neq k$, respectively, as

$$\begin{aligned} \lambda^k + \sum_{j=1}^J \omega^j \frac{\mu_{jk} Q^j}{K (P_s^{*k})^2} e^{-\frac{\mu_{jk} Q^j}{P_s^{*k} K}} &= g_{ss}^k P_s^{*k} + N_0 B + I_{BS}, \\ \frac{g_{ss}^l}{\lambda^l - \delta^{*l}} &= g_{ss}^k P_s^{*k} + N_0 B + I_{BS}. \end{aligned}$$

Consequently, user k is chosen to transmit if $\frac{g_{ss}^k}{\lambda^k} \geq \frac{g_{ss}^l}{\lambda^l}$, $\forall l \neq k$. Let $P_s^{*k} = \max\{\rho, 0\}$, the parameter ρ is obtained from $\rho = \frac{1}{\lambda^k + \varphi(\rho)} - \frac{N_0 B + I_{BS}}{g_{ss}^k}$ where $\varphi(\rho) = \sum_{j=1}^J \omega^j \frac{\mu_{jk} Q^j}{K \rho^2} e^{-\frac{\mu_{jk} Q^j}{\rho K}}$. Finally, updating the Lagrangian multipliers with standard convex programming algorithms, such as the Ellipsoid method, we obtain the optimal values of $\{\lambda^*\}$ and $\{\omega^*\}$.

B. Constrained-Appropriate Maximum Average Transmission Power Adaption (C-AMATPA)

The computational complexity of finding the optimal power allocation in AMATPA is high. This is due mainly to the way the collision probability constraints have been interpreted in (9). We now make more simplifications, called C-AMATPA, to tackle this issue.

Our approach is to develop a looser upper bound of the collision probability constraint. Note that function $w(x) = e^{-\frac{a}{x}}$ for a given positive real number a and $x \geq 0$ is a concave function if $x \geq \frac{a}{2}$. Consequently, considering (9), if $P_s^k(g_{ss})$ satisfies $P_s^k(g_{ss}) \geq \frac{\mu_{jk} Q^j}{2K}$ then $\mathcal{CO}^j \leq \sum_{k=1}^K e^{-\frac{\mu_{jk} Q^j}{\mathbf{E}_{g_{ss}}[P_s^k(g_{ss})]} \frac{Q^j}{K}}$.

The collision probability constraint for the primary receiver j is then reduced to the following two transmission power constraints

$$\sum_{k=1}^K e^{-\frac{\mu_{jk} Q^j}{\mathbf{E}_{g_{ss}}[P_s^k(g_{ss})]} \frac{Q^j}{K}} \leq \xi^j; \quad P_s^k(g_{ss}) \geq \frac{\mu_{jk} Q^j}{2K}, \quad \forall k \in \mathcal{K}. \quad (15)$$

This set of constraints are still difficult to work with. To make the problem mathematically tractable, we simplify the constraint (15) by assuming $e^{-\frac{\mu_{jk} Q^j}{\mathbf{E}_{g_{ss}}[P_s^k(g_{ss})]} \frac{Q^j}{K}} \leq \frac{\xi^j}{K}$, $\forall j \in \mathcal{J}$, $k \in \mathcal{K}$ which means an equal share of collision probability constraint among all secondary transmitters. With some direct manipulations, the collision probability constraints are equivalently interpreted through the following two power allocation constraints:

$$\begin{cases} \mathbf{E}_{g_{ss}}[P_s^k(g_{ss})] \leq \min_j \left\{ \frac{\mu_{jk} Q^j}{K \ln\left(\frac{K}{\xi^j}\right)} \right\} \triangleq \varphi^k \\ P_s^k(g_{ss}) \geq \max_j \left\{ \frac{\mu_{jk} Q^j}{2K} \right\} \triangleq \phi^k, \quad \forall k \in \mathcal{K}. \end{cases}$$

Plugging this in \mathcal{O}_{SM} , instead of the collision probability constraints, the RA problem becomes

$\mathbf{PO}_{SM}^{C-AMATPA}$:

$$C = \max_{\{P_s(g_{ss}) \geq 0\}} \mathbf{E}_{g_{ss}} [C(P(g_{ss}))], \quad (16)$$

$$\text{s.t.} \quad P_s^k(g_{ss}) \geq \phi^k, \quad \forall g_{ss}, \forall k \in \mathcal{K}, \quad (17)$$

$$\mathbf{E}_{g_{ss}} [P_s^k(g_{ss})] \leq \min\{\bar{P}_s^k, \varphi^k\}, \quad \forall k \in \mathcal{K}. \quad (18)$$

This optimization problem is convex, so we can use the dual decomposition method to solve it. Here, we first assume that this optimization problem is feasible. The feasibility condition will be mentioned in the sequel. It can be shown that in each fading realization only one secondary transmitter k transmits with power $P_s^k(g_{ss}) > \phi^k$ and the other transmitters are assigned to transmit with the transmission power ϕ^l , $\forall l \neq k$. The optimal transmission power is then derived as:

$$P_s^k(g_{ss}) = \begin{cases} \max\left\{\phi^k, \frac{1}{\lambda^k} - \frac{N_0 B + I_{BS}}{g_{ss}^k}\right\} & \text{if } g_{ss}^k > \frac{\lambda^k}{\lambda^l} g_{ss}^l, \\ \phi^k & \text{otherwise,} \end{cases} \quad (19)$$

where λ^k s are the Lagrangian multipliers associated with constraints (18). The problem $\mathcal{O}_{SM}^{C-AMATPA}$ is then feasible if $\phi^k \leq \min\{\bar{P}_s^k, \varphi^k\}$ holds $\forall k \in \mathcal{K}$.

C. Appropriate Maximum Peak Transmission Power Adaption (AMPTPA)

AMPTPA operates by exchanging the collision probability constraints with the peak transmission power constraint. This

can be done by developing a new upper bound of the collision probability. To evaluate an upper bound of the collision probability at PS j , the collision probability constraint holds if in each fading realization, we have

$$\mathbf{P} \left\{ \sum_{k=1}^K g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) > Q^j \right\} \leq \xi^j, \quad \forall \mathbf{g}_{ss}, j \in \mathcal{J}. \quad (20)$$

This interpretation of the collision probability constraints actually suggests a conservative way of evaluating this constraint. Eq. (20) is simplified further to $\mathbf{P} \left\{ K \max_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) \leq Q^j \right\} \geq 1 - \xi^j, \quad \forall \mathbf{g}_{ss}, j \in \mathcal{J}$ or equivalently

$$\prod_{k=1}^K \mathbf{P} \left\{ g_{sp}^{jk} \leq \frac{Q^j}{K P_s^k(\mathbf{g}_{ss})} \right\} \geq 1 - \xi^j, \quad \forall \mathbf{g}_{ss}, j \in \mathcal{J}. \quad (21)$$

Condition (21) is guaranteed to be satisfied if

$$\mathbf{P} \left\{ g_{sp}^{jk} \leq \frac{Q^j}{K P_s^k(\mathbf{g}_{ss})} \right\} \geq 1 - e^j \xi^j, \quad \forall \mathbf{g}_{ss}, j \in \mathcal{J}, k \in \mathcal{K}, \quad (22)$$

where e^j 's are constant coefficients equal to $e^j = \frac{1 - \sqrt[K]{1 - \xi^j}}{\xi^j}$ under the assumption of equal sharing of collision probability among all secondary transmitters at each primary receiver. Therefore, (22) is reduced to $\mathbf{P} \left\{ g_{sp}^{jk} \leq \frac{Q^j}{K P_s^k(\mathbf{g}_{ss})} \right\} \geq \sqrt[K]{1 - \xi^j}$ where in the special case of Rayleigh fading fluctuation, this leads to the power allocation constraints $P_s^k(\mathbf{g}_{ss}) \leq -\frac{\mu_{jk} Q^j}{K \ln(1 - \sqrt[K]{1 - \xi^j})}$, $\forall \mathbf{g}_{ss}, j \in \mathcal{J}, k \in \mathcal{K}$. Substituting this in problem \mathcal{O}_{SM} , the RA problem in the case of AMPTPA is reconstructed as

$\mathbf{P}\mathcal{O}_{SM}^{AMPTPA}$:

$$\begin{aligned} C &= \max_{\{P_s(\mathbf{g}_{ss})\}} \mathbf{E}_{\mathbf{g}_{ss}} [C(P(\mathbf{g}_{ss}))], \\ \text{s.t.} \quad (C1), & 0 \leq P_s^k(\mathbf{g}_{ss}) \leq -\frac{\mu_{jk} Q^j}{K \ln(1 - \sqrt[K]{1 - \xi^j})}, \\ & \forall \mathbf{g}_{ss}, j \in \mathcal{J}, k \in \mathcal{K}, \end{aligned}$$

which is convex and its solution can be obtained by following the same lines as in Section III-A2 [14].

Remark 3: IA needs to solve the problem off-line not only to obtain the Lagrange Multipliers but also to evaluate the collision probability constraints for the stopping point of algorithms. On the other hand, AA can actually mitigate this implementation difficulty as follows. First, it need not evaluate the collision probability constraints. Second, according to the recent progress in the area of stochastic resource allocation [27], [28], on-line update of Lagrange multipliers yields the same solution as an off-line scheme. This is possible since the secondary service only needs the accurate CSI between its base-station and users. This is practically significant since as observed in Section V, the performance of AA solution is very close to that of IA, which is very close to optimal.

V. NUMERICAL RESULTS

We use numerical evaluations to confirm the convergence of IA and assess the performance of SS. We would also like

to study the impact of different system parameters, particularly the collision probability constraints, on the ergodic sum capacity of SS (Bits/Sec/Hz). In our simulation study, we set $N_0 B = I_{BS} = 0.1$ Watt and $\forall k, \bar{P}_k^s = 1$ Watt. The fading distribution is Rayleigh.

Remark 4: The collision probability constraint is in general a function of the primary service outage performance. Assuming that primary users do not adopt power allocation, the outage probability of the primary service j is $p_{out}^j = \mathbf{P} \left\{ \frac{g_{pp}^{jj} P_p^j}{N_0 B + I^j + \sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss})} < \gamma^j \right\}$ where γ^j is the SINR threshold and I^j is the interference received from other primary transmissions at the primary receiver j . Note that without spectrum sharing, primary user j still suffers the intrinsic outage due to fading and I^j which is denoted as α_p^j . Let us define $\vartheta^j = e^{-\frac{\gamma^j \mu_{pp}^{jj} N_0 B}{P_p^j}} \mathbf{E}_{I^j} e^{-\frac{\gamma^j \mu_{pp}^{jj} I^j}{P_p^j}}$. Noting $\mathbf{E}X = \int_{t>0} \mathbf{P}(X > t) dt$, we have

$$\begin{aligned} p_{out}^j &= 1 - \vartheta^j \mathbf{E}_{\mathbf{g}_{ss}} e^{-\frac{\gamma^j \mu_{pp}^{jj} \sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss})}{P_p^j}} \\ &= 1 - \vartheta^j \int_{t>0} \mathbf{P} \left\{ e^{-\frac{\gamma^j \mu_{pp}^{jj} \sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss})}{P_p^j}} > t \right\} dt \\ &= 1 - \vartheta^j \int_{0 < t \leq 1} \mathbf{P} \left\{ \sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) < \frac{P_p^j \ln \frac{1}{t}}{\gamma^j \mu_{pp}^{jj}} \right\} dt \\ &= \alpha_p^j + \vartheta^j \int_{0 < t \leq 1} \mathbf{P} \left\{ \sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) \geq \frac{P_p^j \ln \frac{1}{t}}{\gamma^j \mu_{pp}^{jj}} \right\} dt \\ &= \alpha_p^j + \vartheta^j \int_{t \in \tau_-} \mathbf{P} \left\{ \sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) \geq \frac{P_p^j \ln \frac{1}{t}}{\gamma^j \mu_{pp}^{jj}} \right\} dt \\ &\quad + \vartheta^j \int_{t \in \tau_+} \mathbf{P} \left\{ \sum_k g_{sp}^{jk} P_s^k(\mathbf{g}_{ss}) \geq \frac{P_p^j \ln \frac{1}{t}}{\gamma^j \mu_{pp}^{jj}} \right\} dt, \end{aligned}$$

where the set τ_- is defined as $(0, e^{-\frac{\gamma^j \mu_{pp}^{jj} Q^j}{P_p^j}}]$ in which if $t \in \tau_-$ then $Q^j \leq \frac{P_p^j \ln \frac{1}{t}}{\gamma^j \mu_{pp}^{jj}}$. We also define $\tau_+ = \bar{\tau}_-$. Thus, if $t \in \tau_+$ then the inequality $Q^j > \frac{P_p^j \ln \frac{1}{t}}{\gamma^j \mu_{pp}^{jj}}$ holds. Thus, an upper bound on the outage probability is

$$\begin{aligned} p_{out} &\leq \alpha_p^j + \vartheta^j \int_{t \in \tau_-} \xi^j dt + \vartheta^j \int_{t \in \tau_+} dt \\ &= \alpha_p^j + \vartheta^j \left(\xi^j e^{-\frac{\gamma^j \mu_{pp}^{jj} Q^j}{P_p^j}} + 1 - e^{-\frac{\gamma^j \mu_{pp}^{jj} Q^j}{P_p^j}} \right). \end{aligned}$$

Based on the thus-obtained upper bound and outage tolerance of the primary service j , we can choose an appropriate collision probability constraint. A simple numerical evaluation shows that $Q^j = 0.2$ and $\xi^j = 0.1$ may just add 0.2–0.3 on the primary service probability.

Fig. 1 illustrates the convergence pattern of A-IAMTPA for the case of collision probability constraints $(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$. We study the impact of parameter Θ on the speed of convergence. A higher value of Θ leads to faster convergence of A-IAMTPA. For the case of $\Theta = 1$, the

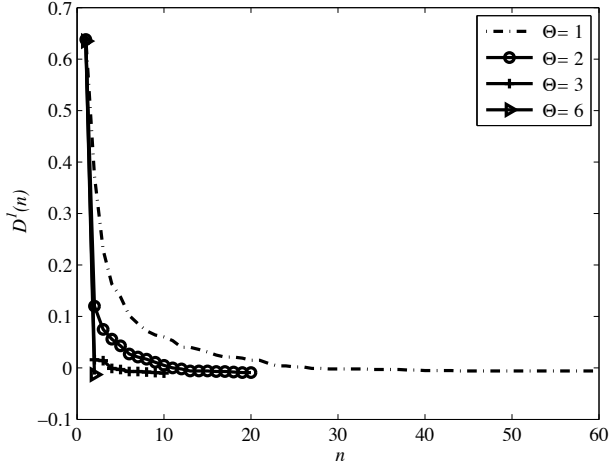


Fig. 1. Convergence of A-IAMTPA versus the number n of iterations for different Θ . Here we set $(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$, and $J = K = 2$.

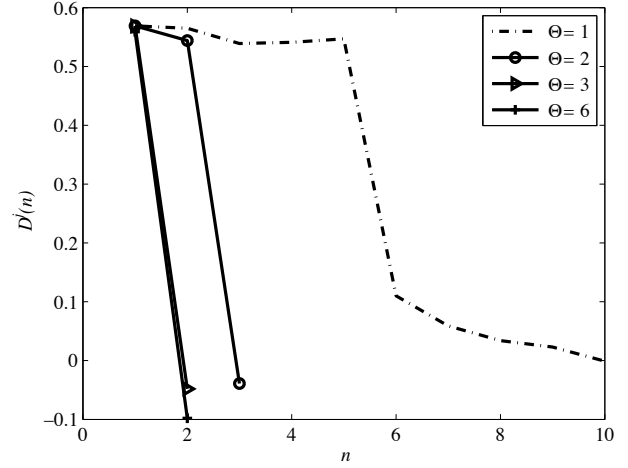


Fig. 3. Convergence of A-IAITA versus the number n of iterations for different Θ . Here we set $(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$, and $J = K = 2$.

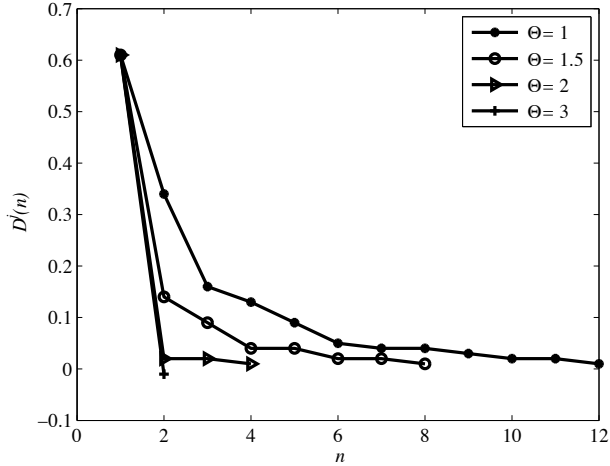


Fig. 2. Convergence of P-IAMTPA versus the number n of iterations for different Θ . Here we set $(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$, and $J = K = 2$.

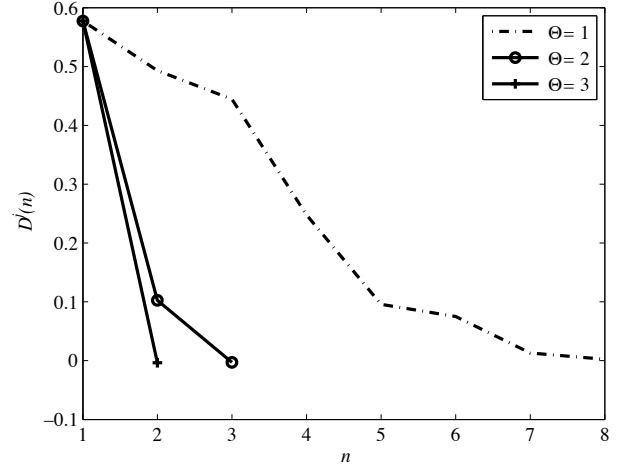


Fig. 4. Convergence of P-IAITA versus the number n of iterations for different Θ . Here we set $(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$, and $J = K = 2$.

algorithm converges after about 60 iterations, but it is quickly reduced to 2 iterations for the case of $\Theta = 6$.

Fig. 2 shows the impact of Θ on the convergence of P-IAMTPA ($(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$). In this case, like A-IAMTPA, the speed of convergence is increased by increasing Θ . For example, in the case of $\Theta = 1$ after 12 iterations, we get the true collision probability constraint $\xi = 0.1$. Comparing this figure with Fig. 1, in P-IAMTPA the algorithm converges almost 5 times faster than A-IAMTPA. This is due mainly to the fact that in P-IAMTPA the auxiliary peak transmission power constraints squeeze the transmission power of users faster than the average ones in A-IAMTPA.

Fig. 3 shows the impact of Θ on the convergence of A-IAITA ($(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$). Increasing Θ is shown to increase the convergence speed. Furthermore, the rate of convergence in A-IAITA is higher than both A-IAMTPA and P-IAMTPA. In fact, for $\Theta = 1$ in A-IAITA after 10 iterations appropriate collision probabilities are obtained. These numbers

in A-IAMTPA and P-IAMTPA are 60 and 12, a direct result of exploiting the average interference threshold constraint in P-IAMTPA.

Fig. 4 shows the convergence of P-IAITA for different values of parameter Θ . Here, we again set $(Q^j, \xi^j) = (0.1, 0.1)$, $\forall j$. P-IAITA is shown to converge very fast even for the case of $\Theta = 1$. One can easily observe that P-IAITA outperforms the other cases in convergence speed.

Fig. 5 shows the total number of iterations in AA versus the interference threshold constraint when $\Theta = 1$. Increasing Q is shown to decrease the iteration count, as we expected. When Q is high enough, a larger transmission power may not violate the collision probability constraints.

In general, IAITA converges faster than IAMTPA. Furthermore, in both cases, approaches involving peak versions of transmission power and interference threshold constraints (P-IAMTPA and P-IAITA) converge faster than average transmission power and interference threshold constraints (A-IAMTPA

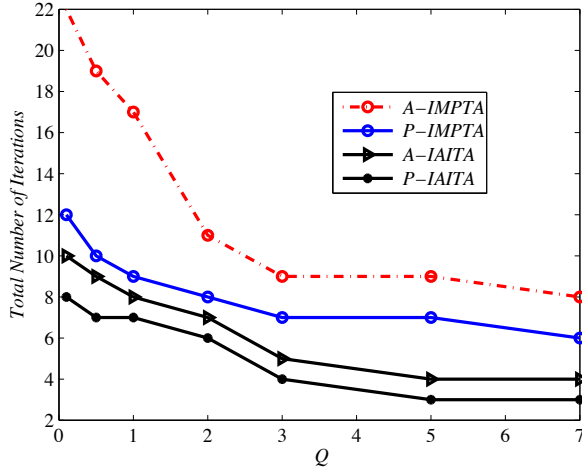


Fig. 5. Total number of iterations in IA versus $Q^j = Q, \forall j, J = K = 2$, and $\xi = 0.1$.

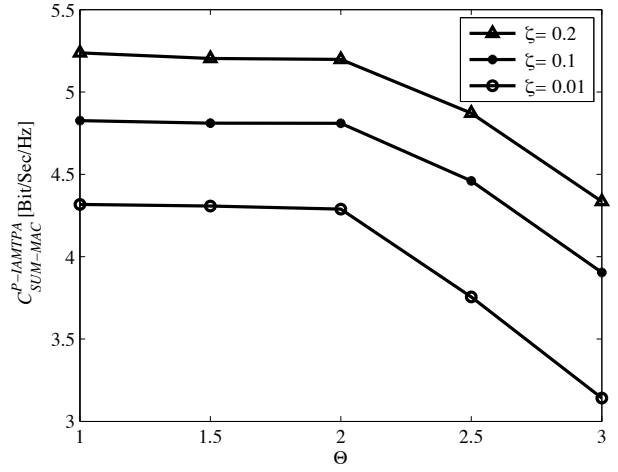


Fig. 7. Ergodic sum capacity of SS in P-IAMTPA versus Θ for different maximum allowable collisions $\xi^j = \xi, \forall j$. Here we set $Q^j = 0.1, \forall j$, and $J = K = 2$.

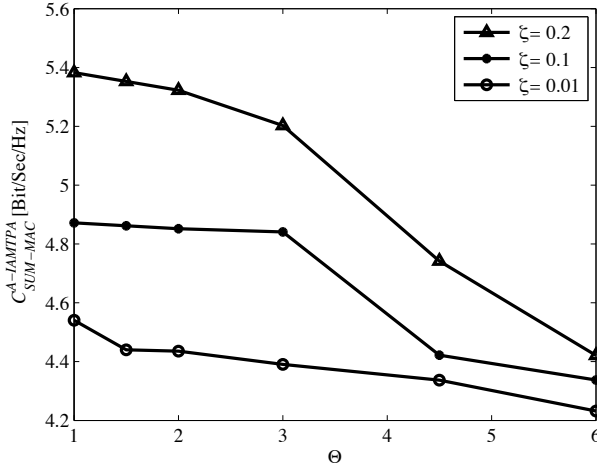


Fig. 6. Ergodic sum capacity of SS in A-IAMTPA versus Θ for different maximum allowable collisions $\xi^j = \xi, \forall j$. Here we set $Q^j = 0.1, \forall j$, and $J = K = 2$.

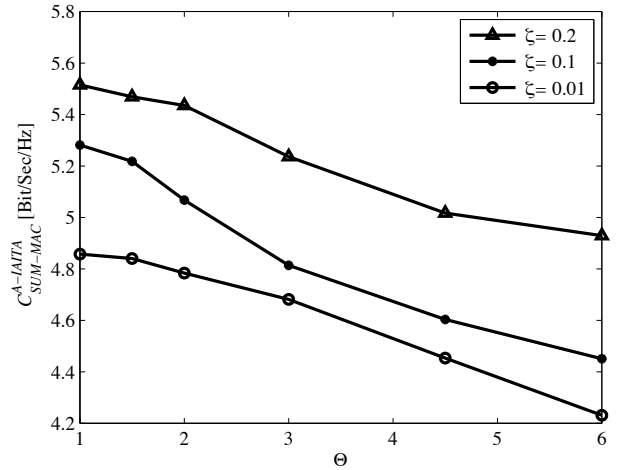


Fig. 8. Ergodic sum capacity of SS in A-IAITA versus Θ for different maximum allowable collisions $\xi^j = \xi, \forall j$. Here we set $Q^j = 0.1, \forall j$, and $J = K = 2$.

and A-IAITA). Fig. 6 shows $C_{SM}^{A-IAMTPA}$ vs. Θ for different values of $\xi = \xi^j, \forall j$. Obviously, a larger ξ results in a larger ergodic sum capacity. Lower values of ξ lower the transmission power of secondary users, thus reducing their performance. Furthermore, increasing Θ decreases $C_{SM}^{A-IAMTPA}$. A larger Θ limits the power allocated to the secondary users. Comparing Figs. 6 and 1, we conclude that a larger Θ leads to faster convergence at the expense of lower performance of SS.

Fig. 7 illustrates the ergodic sum capacity of SS in P-IAMTPA vs. Θ where we also study the impact of ξ on $C_{SM}^{P-IAMTPA}$. The larger ξ , the larger $C_{SM}^{P-IAMTPA}$. Moreover, increasing Θ decreases $C_{SM}^{P-IAMTPA}$. Comparison of Fig. 7 with Fig. 6 indicates the fact that in P-IAMTPA Θ has more pronounced effects on ergodic sum capacity due to auxiliary peak transmission power constraints.

We now study the impact of Θ and ξ on $C_{SM}^{A-IAITA}$ and $C_{SM}^{P-IAITA}$ in Figs. 8 and 9, respectively. The same pattern

as in both Figs. 7 and 6 can again be obtained for A-IAITA and P-IAITA. By comparing the results in Figs. 6, 7, 8, and 9, we can conclude that $C_{SM}^{A-IAMTPA} \geq C_{SM}^{P-IAMTPA}$ and $C_{SM}^{A-IAITA} \geq C_{SM}^{P-IAITA}$. This is due mainly to the fact that in A-IAMTPA (A-IAITA) we employ the average transmission power constraint (the interference threshold constraint) which contains the allocated power in the case of P-IAMTPA (P-IAITA).

The ergodic sum capacity of SS in C-AMATPA is plotted in Fig. 10, showing the impacts of K and Q . In this case, increasing K may not increase $C_{SM}^{C-AMATPA}$. If the multiuser diversity is dominant, $C_{SM}^{C-AMATPA}$ will be increased by increasing K . However, since a large K limits the impact of multiuser diversity due to the constraint (17) and also due to the way the parameters ϕ^k are defined, a larger K makes ϕ^k smaller and the SS performance lower. Similar to C-AMATPA, we can also evaluate capacity for AMPTPA and AMATPA but

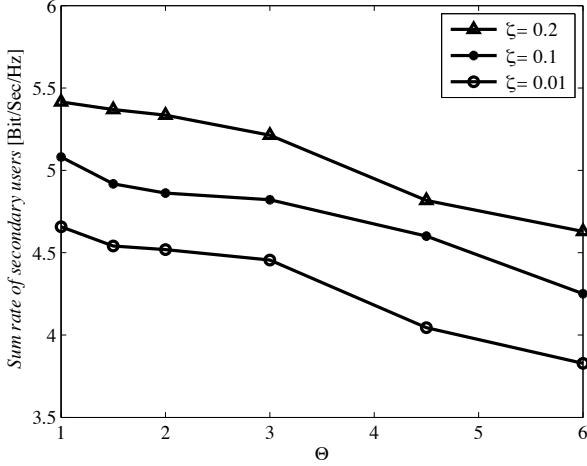


Fig. 9. Ergodic sum capacity of SS in P-IAITA versus Θ for different maximum allowable collisions $\xi^j = \xi, \forall j$. Here we set $Q^j = 0.1, \forall j$, and $J = K = 2$.

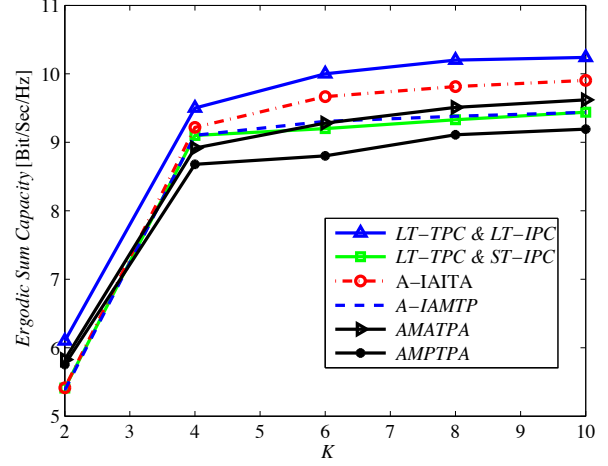


Fig. 11. Ergodic sum capacity of SS for IA and AA versus K . Here we set $(Q^j, \xi^j) = (0.1, 0.2), \forall j$, and $J = 2$.

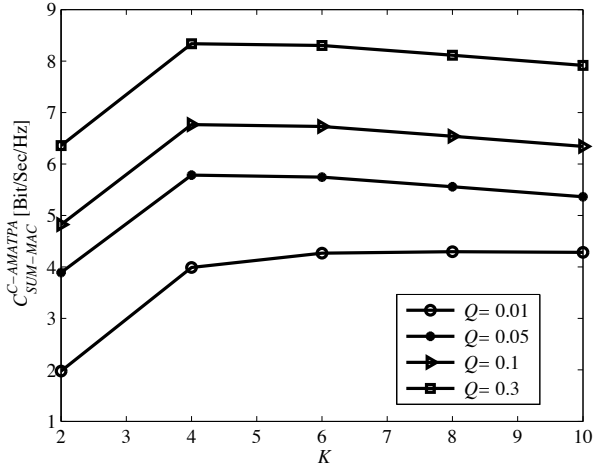


Fig. 10. Ergodic sum capacity of SS in C-AMATPA versus K for different interference threshold constraint $Q^j = Q, \forall j$. Here we set $\xi^j = 0.2, \forall j$, and $J = 2$.

omit it due to page limit. Note that in C-AMATPA, AMPTPA, and AMATPA increasing Q will result in increasing capacity which is in line with previous findings in [9].

Fig. 11 compares the performance of IA and AA vs. the number of secondary users K , showing that in almost all cases, IA yields higher performance than AA. Table II also compares the performances of IA and AA for different values of ξ , which is consistent with the result of Fig. 11. Since collisions happen due to the aggregated interference at the primary receiver, for a more thorough evaluation of the collision probability, we need a cooperative framework between secondary users. Fig. 11 also shows $C_{SM}^{A-IAITA} \geq C_{SM}^{A-IAMTP}$ since in A-IAITA for selection of users, we should consider two different vector channel power gains, thus exploiting a higher multi-user diversity gain. Here we also show the performance of SS assuming the availability of perfect CSI g_{sp}^{jk} in [14]. Two different scenarios are considered: LT-TPC LT-IPC (scenario

TABLE I
COMPUTATIONAL COMPLEXITY

Scheme	Computational Complexity
LT-TPC LT-IPC	$\mathcal{O}((J + K)^2)$
LT-TPC ST-IPC	$\mathcal{O}((K\mathcal{O}(J^2))^2)$
A-IAITA	$\varrho \sqrt{L_{A-IAITA}^{max}} [\mathcal{O}((J + K)^2) + JC_{MC}]$
P-IAITA	$\varrho \sqrt{L_{P-IAITA}^{max}} [\mathcal{O}((K\mathcal{O}(J^2))^2) + JC_{MC}]$
A-IAMTP	$\varrho \sqrt{L_{A-IAMTP}^{max}} [\mathcal{O}(K^2) + JC_{MC}]$
P-IAMTP	$\varrho \sqrt{L_{P-IAMTP}^{max}} [\mathcal{O}((K\mathcal{O}(K^2))^2) + JC_{MC}]$
AMATPA	$\mathcal{O}((J + K)^2)$
AMPTPA	$\mathcal{O}((K\mathcal{O}(K^2))^2)$
C-AMATPA	$\mathcal{O}(K^2)$

I which is Problem 3.1 in [14]) and LT-TPC ST-IPC (scenario II which is Problem 3.3 in [14]) where in scenario I (scenario II) long-term (short-term) interference threshold constraints are considered. Scenario I is shown to outperform scenario II and our results in this paper. Note that here we set $\xi^j = \xi = 0.2$. From Table II, we observe that for $\xi = 0.3$ A-IAITA outperforms the performance of scenario I. For a large enough collision probability, the secondary service is able to transmit with higher power without harming the primary service performance.

Table III shows the special case of $J = K = 1$ for $Q = 0.1$. Here we can obtain the optimal solution for the case of Rayleigh fading as mentioned in Remark 1. The thus-obtained capacity based on IA is very close to optimal. For example, in the case of A-IAITA, the difference is only about 0.09. This result also indicates that A-IAITA can accurately estimate the SS performance.

Finally, Table I shows the computational complexities of IA and AA, where $L_{A-IAITA}^{max}$ stands for the number of iterations that can be obtained from Fig. 5. For other IA schemes, the associated parameter is similarly defined. From the simulation results, we also found that the number of iterations is actually reduced by increasing Θ , which was incorporated in Table I. Also, the complexity due to the numerical evaluation of colli-

TABLE II
COMPARISON OF ERGODIC CAPACITY OF DIFFERENT SCHEMES ($Q = 0.1, J = 2, K = 6$).

ζ	LT-TPC/LT(ST)-IPC	A-IAITA	P-IAITA	A-IAMTP	P-IAMTP	AMATPA	AMPTPA	C-AMATPA
0.01	10.0013 (9.20124)	8.27099	7.92213	7.7332	6.8123	7.63041	6.9562	3.7321
0.2	10.0013 (9.20124)	9.6677	9.45125	9.3067	9.0123	9.28014	9.1919	6.7546
0.3	10.0013 (9.20124)	11.8124	11.2485	11.1394	10.68354	10.9954	10.7965	8.2138

TABLE III
SPECIAL CASE OF $J = K = 1$ WITH $Q = 0.1$.

ζ	Optimal	A-IAITA	P-IAITA	A-IAMTP	P-IAMTP
0.01	4.0234	3.9945	3.6146	3.2368	3.035
0.1	4.6340	4.6120	4.5012	4.485	4.2123
0.2	5.1956	5.1127	5.0248	4.9687	4.78354

sion probability constraints adopting Monte Carlo simulation is taken into account via parameter C_{MC} . Of IA schemes, A-IAMTP has the lowest computational complexity. Also, C-AMATPA has the lowest computational complexity in AA. Furthermore, the complexity of C-AMATPA is lower than A-IAMTP.

VI. CONCLUSIONS AND DISCUSSIONS

In this paper the RA problem in the uplink of a spectrum-sharing network was investigated. We assumed that the SS only knows the channel distribution information (CDI) between its transmitters and the primary receivers. We introduced the concept of *collision probability constraints* to deal with the PS's unexpected QoS degradation due to SS activities. Iterative Approach (IA) and Analytical Approach (AA) were then used to find suboptimal solutions. IA further divided in four different categories: Average-Iterative Appropriate Maximum Transmission Power Adaption (A-IAMTPA), Peak-Iterative Appropriate Maximum Transmission Power Adaption (P-IAMTPA), Average-Iterative Appropriate Interference Threshold Adaption (A-IAITA), and Peak-Iterative Appropriate Interference Threshold Adaption (P-IAITA). We proved that all of the aforementioned algorithms converge to suitable transmission power sets. Using corroborating simulation results, these algorithms are shown to converge quickly. We also observed that both A-IAITA and P-IAITA converge faster than A-IAMTPA and P-IAMTPA as we substituted the collision probability constraints with different versions of interference threshold constraints. P-IAITA (P-IAMTPA) was also found to converge faster than A-IAITA (A-IAMTPA) mainly because the former further restricted the power of the secondary transmitters. Previous findings in [9], [10] indicate that the former would gain a smaller capacity than the latter. We also observed the same trend. We also introduced the control parameter $\Theta \geq 1$: by changing its value, one can make a tradeoff between the speed of convergence and the resulting ergodic sum capacity: a larger Θ results in quicker convergence at the expense of lower ergodic sum capacity.

In the case of AA, we derived some mathematically suitable versions of the collision probability constraints by inferring them as peak/average transmission power constraints. AA is divided further into three more categories: Appropri-

ate Maximum Average Transmission Power Adaption (AMATPA), Constrained-Appropriate Maximum Average Transmission Power Adaption (C-AMATPA), and Appropriate Maximum Peak Transmission Power Adaption (AMPTPA). In all of these categories, each secondary transmitter independently construed the collision probability constraints into its own suitable peak/average transmission power constraints. Thus, unlike IA, we did not need extra signaling between the secondary transmitters and the secondary base-station to adjust appropriate power/interference constraints. Nevertheless, this implementation caused degradation of SS spectral-efficiency.

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Mohammad G. Khoshkholgh received his B.Sc. degree in electrical engineering from Isfahan University, Isfahan, Iran, in 2006, and his M.Sc. degree in electrical engineering from the Tarbiat Modares University, Tehran, Iran, in 2008. He was with the Wireless Innovation Laboratory at Tarbiat Modares University from 2008 until 2012. He is now with the Simula Research Laboratory, Fornebu, Norway. His research interests are mainly in wireless communications, radio resource allocations, and spectrum sharing.



Nader Mokari Yamchi is currently working toward the Ph.D. degree at Tarbiat Modares University, Tehran, Iran. His research interests include wireless communications, radio resource allocations, cooperative, network planning, and spectrum sharing. He is a student member of IEEE.



Kang G. Shin is the Kevin & Nancy O'Connor Professor of Computer Science in the Department of Electrical Engineering and Computer Science, The University of Michigan, Ann Arbor. His current research focuses on QoS-sensitive computing and networking, as well as embedded real-time and cyber-physical systems.

He has supervised the completion of 73 Ph.D.s, and authored/co-authored more than 780 technical articles (more than 300 of these in archival journals), one textbook, more than 20 patents or invention disclosures, and has received numerous best paper awards, including the Best Paper Awards from the 2011 ACM International Conference on Mobile Computing and Networking (MobiCom'11), the 2011 IEEE International Conference on Autonomic Computing, the 2010 and 2000 USENIX Annual Technical Conferences, as well as the 2003 IEEE Communications Society William R. Bennett Prize Paper Award and the 1987 Outstanding IEEE TRANSACTIONS ON AUTOMATIC CONTROL Paper Award. He has also received several institutional awards, including the Research Excellence Award in 1989, the Outstanding Achievement Award in 1999, the Distinguished Faculty Achievement Award in 2001, and the Stephen Attwood Award in 2004 from The University of Michigan (the highest honor bestowed to Michigan engineering faculty); a Distinguished Alumni Award of the College of Engineering, Seoul National University in 2002; the 2003 IEEE RTC Technical Achievement Award; and the 2006 Ho-Am Prize in Engineering (the highest honor bestowed to Korean-origin engineers).

He has chaired several major conferences, including 2009 ACM MobiCom, 2008 IEEE SECON, 2005 ACM/USENIX MobiSys, 2000 IEEE RTAS, and 1987 IEEE RTSS. He is a fellow of both IEEE and ACM, and served on editorial boards, including the IEEE TPDS and the *ACM Transactions on Embedded Systems*. He has also served or is serving on numerous government committees, such as the US NSF Cyber-Physical Systems Executive Committee and the Korean Government R & D Strategy Advisory Committee. He has also co-founded a couple of startups.